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Retinal Vessel Extraction Using a Piecewise Gaussian Scaled Model

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Abstract—**Automated analysis of retinal images usually requires estimating the positions and appearance of blood vessels, which contain important features for abnormality detection. Although there is a wide literature on detecting vessel positions from retinal images by modelling cross-sectional profiles, little attention has been given to extracting vessel appearance in intensity. In this paper, we introduce a piecewise Gaussian scaled model to characterise the intensity distributions of vessel cross-sections. Based on a newly developed vessel detection scheme, we describe the use of the proposed model for extracting vessel appearance. The preliminary experimental results obtained from angiographic pairs and images of an SLO sequence are reported.**

I. Introduction

IMAGES of the ocular fundus have a wide range of applications in clinical practice, including the diagnosis of diabetic retinopathy, retinal densitometry, and macular pigment measurements [1]. The appearance of blood vessels in ocular fundus is a central feature for diagnosis support, since it can be used to classify veins and arteries, measure diameter changes, and visualise retinal microcirculation [2] [3] [4]. For example, retinal vessels can be classified into arteries and veins based on their appearance in intensity, and the ratio of arteriole and venular vessels in retinal images is regarded as an indication of cardiovascular diseases [2]. Another example is the appearance of vessels in angiographic sequences which varies significantly in retinal microcirculation so that a fused image can be generated to illustrate circulation at all points in the vascular network. This enables identifying retinal vascular occlusions that may occur during cardiopulmonary bypass surgery [4]. A precise characterisation of vessel appearance in retinal images of multiple modalities is therefore of great importance.

Vessels are objects with a two-sided boundary, while the "twin" boundaries of vessel are often assumed to run smoothly or parallel to each other. The task of vessel detection can be accomplished by convolving images with a filter kernel defined by the model of cross-sectional profile along the vessel longitude (e.g., Gaussian shaped profile or difference-of-dual Gaussians) [5] [6]. There is a wide literature on parametric profile models characterising the appearance of retinal vessels. It is a natural choice to extract the appearance by optimally finding model parameter values.

In practice, a vessel cross-section is often partitioned into three segments corresponding to the two boundaries and the central between the boundaries, so as to account for the central reflex [7] [8]. Li *et al.* proposed to model each of three segments as part of a Gaussian curve, assuming that the intensity background of twin boundaries remains at the same level [7]. Their piecewise Gaussian model has been shown to be effective in differentiating arteriole and venular vessels in colour fundus images based on the brightness of vessels.

Retinal vessels are visible as dark or bright structures relative to the background (dark in red-free photographic images, bright in many fluorescein angiograms). Since dye injections help to highlight the vascular tree, the brightness of vessels in an angiographic sequence of SLO (Scanning Laser Ophthalmoscope) images increases from low to a peak, then decreases to original low values; that is, vessels change from dark to bright, then back to dark (see Fig. 1). It is therefore difficult to fit a piecewise model to vessel segments, especially in SLO sequences.

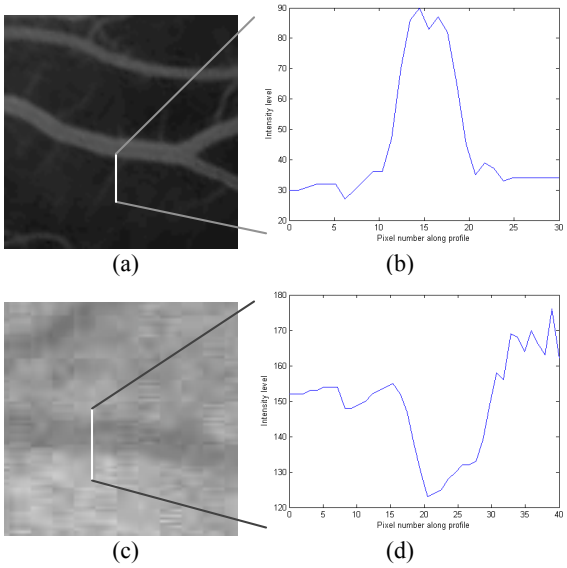

Fig. 1. Cross-section of a bright vessel (a) and its intensity profile (b); the cross-section of a dark vessel (c) and its intensity profile (d). The image patches (a) and (c) are cropped from two angiographic images respectively.

In the rest of this paper, a piecewise Gaussian scaled model is proposed to better characterise the intensity distribution crossing vessels. We also discuss how to use the model obtained to extract vessel appearance, based on a newly developed vessel detection scheme.

II. Methodology

A. Modelling Vessel Profile

As to the intensity distribution on the cross-section, we

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make the following remarks:

- The background intensity on the twin boundary edges may be significantly different.
- Vessels in a sequence may change between dark and bright, relative to the background.

To better describe the intensity distribution of vessel, we propose a piecewise Gaussian scaled model:

$$y = f(x) = \begin{cases} -\gamma_1 e^{-\frac{(x-p_1)^2}{2\sigma_1^2}} + I_1 & x < p_l \\ \gamma_2 e^{-\frac{(x-p_2)^2}{2\sigma_2^2}} + I_2 & p_l \le x \le p_r \\ -\gamma_3 e^{-\frac{(x-p_2)^2}{2\sigma_3^2}} + I_3 & x > p_r \end{cases}. \quad (1)$$

which consists of three Gaussian functions, depicting the curves of the twin boundaries and the central curve with or without the reflex, respectively. As illustrated in Fig. 2, γ_1, γ_2, and γ_3 represent the amplitudes of the three curves; p_1, p_2, and p_3 are the positions of the peaks of the Gaussian functions; σ_1, σ_2, and σ_3 denote width distributions; I_1 and I_3 are the intensities of retinal background of the twin boundaries, and I_2 is the minimum intensity level of the central curve.

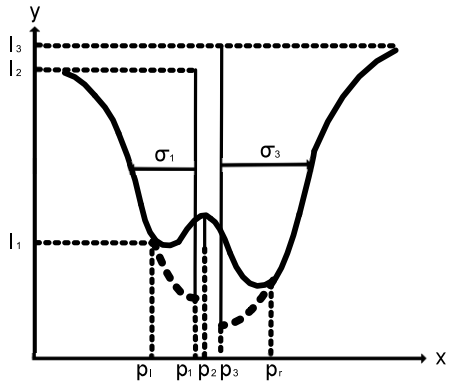


Fig. 2. Cross-sectional profile of vessel.

B. Vessel Extraction

We aim to determine the amplitude of Gaussians on each piecewise curve. However, it is non-trivial to determine these curves on a cross-section. In the following, we discuss some properties of Gaussian scaled functions, and describe the use of the proposed piecewise Gaussian scaled model for vessel extraction.

1) Properties of Gaussian Scaled Functions: Without loss of generality, let us consider that a random vector A can be approximated by the product of a zero-mean Gaussian vector H and an independent positive scalar random variable γ:

$$\mathrm{A} \overset{d}{=} \gamma \mathrm{H} \quad (2)$$

where $\overset{d}{=}$ indicates equality in distribution [9]. Suppose the amplitudes of A are corrupted by additive noise:

$$\mathrm{Y} = \mathrm{A} + W_n,$$

where W_n is zero-mean Gaussian noise, and Y is the observed vector. Without loss of generality, the vector Y is expressed as:

$$\mathrm{Y} = \pm\gamma\mathrm{H} + W_n \quad (3)$$

where "+" or "-" is dependent on upward or downward signals (i.e., bright or dark object relative to background). As an approximation, H and W_n are assumed to be decorrelated.

Let us recall that the distribution of signals for the classical matched filtering is assumed Gaussian. Given the values of the multiplier γ, a matched filter constructed with the shape of H can then be used to detect the signals (i.e., vector A) with scaled Gaussian. In practice, values of the multiplier γ are unknown beforehand, and the multiplier is sought by solving an optimisation problem. We have an initial estimator for the multiplier:

$$\hat{\gamma}(\mathrm{Y}) = \frac{\mathrm{Y}^T\mathrm{Y}}{N} - \sigma_\omega^2, \quad (4)$$

where N is the dimensionality of vectors A, H, W_n, and Y. An estimate of the multiplier is adjusted based on the initial estimator (4) until the maximal SNR of the fitting is approximated.

2) Multiscale Analysis for Vessel Detection: The Fourier transform of a Gaussian function is constant zero-phase, and the phase of a Difference-of-dual-Gaussians switches being zero and π. Both exhibit scale invariance (i.e., phase invariance with respect to scales) in the wavelet domain. With this observation, we approach the task of vessel detection by assessing Fourier phase alignment. Considering that the piecewise Gaussian scaled model in (2) holds scale invariance, we extend the idea of phase alignment for vessel extraction by implementing a multi-scale analytical scheme. First, on the one hand, detection of vessel positions is performed by calculating phase alignment with zero or π (for dark or bright vessels), or with switching zero and π (for vessels with the central reflex). The positions with high alignment coincide with the vessel median area delineating the vascular structure. On the other hand, phase alignment with $\pm\pi/2$ is calculated and the positions with high alignment coincide with vessel boundary areas. Details of calculating phase alignment can be seen in [8]. Consequently, the twin boundary and the central of a cross-section are determined.

3) Fitting Model to Data: The merit function that measures the agreement between model and data is defined as the Euclidean distance,

$$e = \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2 \quad (5)$$

where (x_i, y_i) is the i-th sampling point, and n is the number of the data points. Best-fit parameters can be obtained by minimising the merit function in (5). For this

purpose the results reported in this paper were obtained using the Marquardt method.

C. Summary of Algorithm

The cross-sectional profile of vessels is modelled as the product of an independent scalar random variable γ and a Gaussian shaped function H. The scalar variable of the model implies the appearance of a vessel cross-section. Hence, the values of γ extracted from the same cross-section across an angiographic sequence, reflect the interval variation of vessel brightness caused by dye injection. We apply the phase alignment function on data from oriented 2-D log-Gabor wavelets. The computation of phase alignment and extraction of cross-sections on 2-D images follow the implementation of [8]. Empirically we chose six orientations with four resolution levels increasing the wavelength by 0.2 octaves.

The algorithm hence proceedings in the following steps:

- Calculate phase alignment with zero or π on each image producing a map of the vascular structure.
- Calculate phase alignment to produce a map of boundary areas.
- Extract cross-sections.
- Perform optimisation to find the value of the multiplier on each vessel cross-section.

III. Experimental Results

A pair of cropped FA images acquired from the same subject at different time are given in Figs 3(a) and 3(c), and their zero-phase detection results are given in Figs 3(b) and 3(d) respectively. In the original FA images, the central light reflex due to imaging or physiologic conditions of the subject is obvious on some vessel segments. Careful observation of the original image in Fig. 3(c) shows that a few vessel segments are dark relative to the surrounding background, while other vessel segments are bright. The alignment degrees shown in Figs 3(b) and 3(d) take values between 0 and 1. From the results we readily observe that high alignment degrees of local Fourier components are detected coinciding with the positions of the vasculature trees. Observation of the result in Fig. 3(b) also supports that high alignment degrees are retained even if the central light reflex is present.

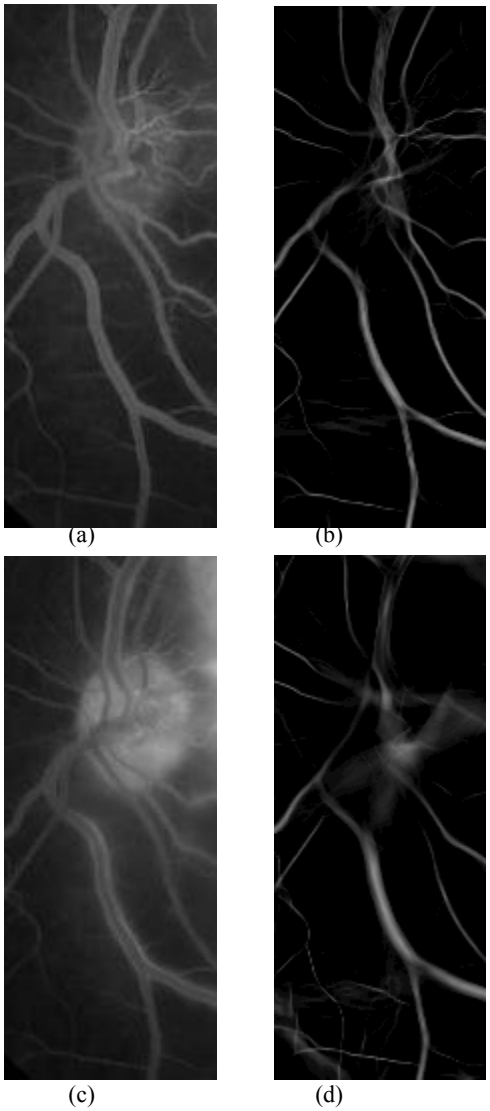
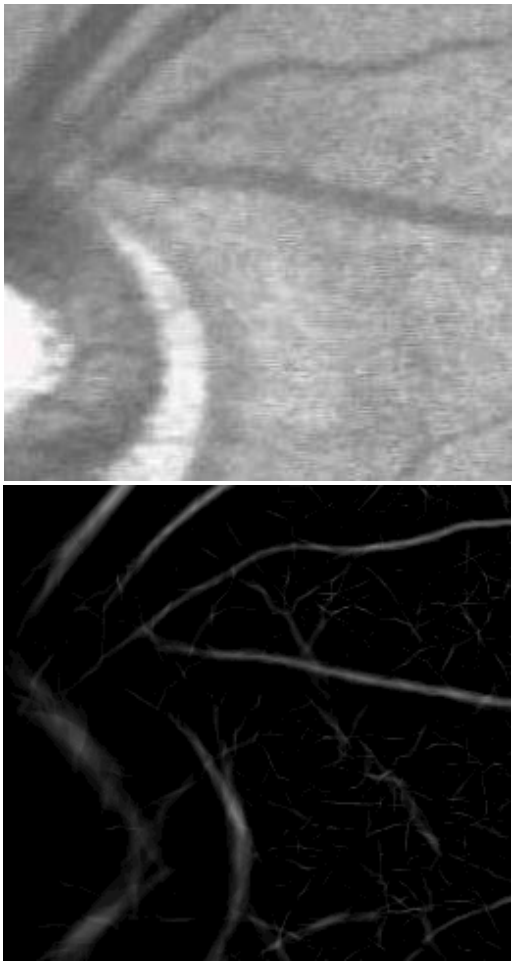


Fig. 3. FA images (a) and (c) acquired from one subject at different time; and their detection results (b) and (d) of zero phase.

A cropped original SLO image is given in Fig. 4(a), and the detection result of zero-phase in Fig. 4(b). For the purpose of comparison, the classical matched filtering ([5] and [10]) was applied to the same image, and the corresponding filtered image is given in Fig. 4(c). We notice that the classical matched filtering is ineffective, largely because the SLO image is fairly noisy. By contrast, the result of zero-phase detection delineates the vasculature tree. This shows that the method of zero-phase detection is hence more noise robust.



(a) (b)

Fig. 4. A cropped SLO image (a) and the detection result of zero phase (b). The corresponding detection result (c) of the Gaussian matched filtering.

The fitting experiment was performed on the cross-section illustrated in Fig. 1(c), across angiographic images of an SLO sequence. The material of this experiment includes 35 images in time order taken from the frames of the sequence which have been registered beforehand. We used the Levenberg-Marquardt algorithm to optimise values for the model multiplier. These values were recorded and plotted in Fig. 5, reflecting the dye filling progress. We can see the proposed technique provides an interface to illustrate the dye filling progress, i.e., the time to peak.

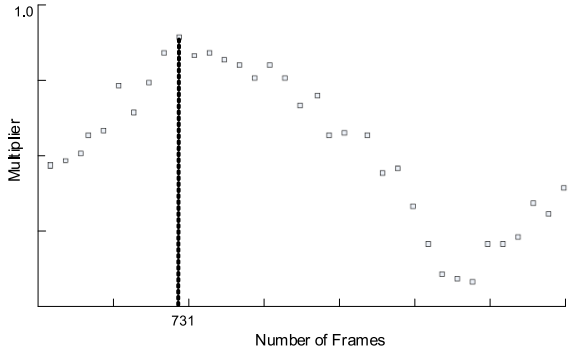


Fig. 5. Multiplier at the cross-section as shown in Fig 1(c), calculated from SLO sequence.

IV. Discussion

We have presented a new vessel extraction technique for the understanding of FA pairs and sequences. The novelty of this work is that a piecewise Gaussian scaled function for modelling the distribution of vessel cross-sections is proposed, and the properties of the Gaussian scaled function are utilised for extraction. The proposed algorithm technique retains the advantage of a newly developed scheme, and has following strengths attractive for image sequences:

- dark and bright vessels with varying appearance can be detected;
- appearance changes can be extracted;
- noise-robustness;
- minimal operator intervention partly due to phase congruency.

Presently, we are assessing the proposed algorithm technique on more SLO sequences, and expect to refine it for cases where both dark and bright vessels are present simultaneously in single images. The driving application is super-resolution of angiographic images of a SLO sequence acquired during retinal circulation.

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