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# A Novel Shape Representation of the Dental Arch and its Applications in Some Dentistry Problems

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***Abstract*— Standardized digital images of maxillary dental casts of 47 subjects were analyzed using MATLAB software whereby the two hamular notches and the incisive papilla defines the Cartesian vertical and horizontal axes, as well as the origin. The angle and length of the midpoints of the anterior teeth, mesiobuccal and distobuccal cusp of the posterior teeth were measured from the origin and denoted as $\theta_1,\ldots,\theta_{18}$ and $l_1,\ldots,l_{18}$ respectively. These values were collectively used to represent the shape of each dental cast. Clustering and principal component analyses were employed to find possible groups of dental arches using the above measure of shape. The main result of this study is that the 3 groups of dental arch shape may be represented by the novel feature vector $\underline{\bar{v}_k} = (\bar{\theta}_1^k, \bar{l}_1^k, \bar{\theta}_3^k, \bar{l}_3^k, \bar{\theta}_5^k, \bar{l}_5^k, \bar{\theta}_{13}^k, \bar{l}_{13}^k), k = 1,2,3$. Knowledge of $\underline{\bar{v}_k}$ implies three impression trays should be sufficient in a particular prosthetic dentistry application for Malaysian patients. Further, given that $\underline{\bar{v}_k}$ are accurately measured, they may be potential candidates as evidence in specific application of forensic dentistry.**

## I. Introduction

THE shape and size of the dental arch has been the subjects of discussion and research in dentistry since early 1880. One of the areas of interest in determining the arch shape arose from clinical experience in finding suitable impression trays that fits a dentate patient for impression making [1],[2]. Impression trays are generally U-shaped and multiple sizes are made to accommodate a range of mouth sizes. Though various choices of stock trays are available commercially, the problem of finding an impression tray that perfectly fits a patient still remains [1],[3]. Some trays are often too long or too short in relation to the extent of the oral soft and hard tissues that need to be recorded and modifications to the trays need to be carried out before the trays can be selected for making acceptable impressions. As for the inexperienced dental practitioners, selecting the right impression tray may be challenging and providing a few trays that generally accommodate a specific population would make the selection process easier.

From a review of the literature, there appears to be no comprehensive study of stock tray designs in relation to the size and shape of the human dental arch. Therefore, describing shape and size of dental arches would be the first step in achieving a scientific design of stock trays.

In earlier studies, the shape of the human dental arches had been qualitatively described as semi-ellipse, parabolic curve, and catenary curve [4]-[6]. Then, a more sophisticated representation of shape had been proposed by fitting polynomial equations, conic section, and beta function [7]-[9]. This study is aimed at exploring a novel method of representing the dental arch shape in relation to stock tray design. Knowledge of $\underline{v} = (\theta_1, l_1, \theta_3, l_3, \theta_5, l_5, \theta_{13}, l_{13})$ and groups of arch shape also has an important application in prosthetic dentistry (treatment of edentulous patient) and forensic dentistry [7], [10]-[15].

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## II. Materials and Methods

### A. Data Selection and Stone Cast Making

Forty seven dentate Malaysian adults aged between 19 and 32 years with well aligned maxillary anterior teeth and minimal attrition participated in this study. Subjects were excluded if they had a history of orthodontic treatment, anterior restoration or a fixed dental prosthesis in the maxilla or mandible. Impressions of dental and oral structures of the maxillary arches of the subjects were made with irreversible hydrocolloid (Duplast fast set alginate impression material; Dentsply Dental Co Ltd, Tianjin, China). Impressions were then cast using type III dental stone (Moldano; Heraeus Kulzer GmbH, Hanau, Germany).

### B. *Digitized Dental Cast and Image Registration*

Standardized digital images from the occlusal surfaces of the stone casts were captured by a high resolution digital camera (Nikon D70s; Nikon Corp, Tokyo, Japan). The camera-object distance was fixed at approximately 50 cm to ensure distortion-free images. Two metal rulers fixed perpendicular to each other and positioned on a plane parallel to the occlusal plane were used as frames for calibration of the measurements (Fig. 1). These images were then saved as JPEG files.

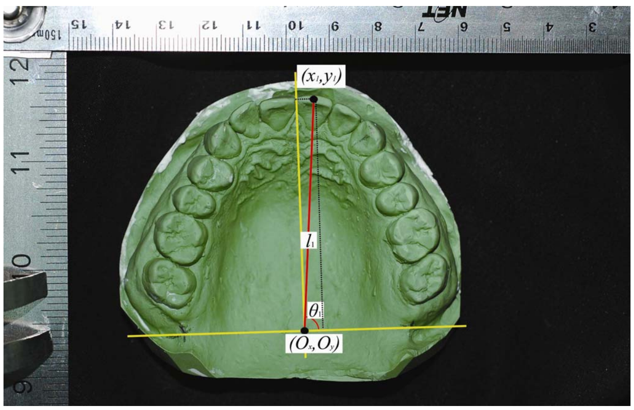

Fig. 1. Computation of angle and distance of the tooth from the geometrical Cartesian origin made on digital images of casts positioned and calibrated in a standardized manner using two metal rulers positioned on a plane parallel to the occlusal plane.

A program was developed in MATLAB software (version R2009b, The MathWorks Inc., USA) so that all the measurements in reference to the anatomical landmarks (incisive papilla, right and left hamular notches) can be conveniently measured [16], [17]. The line joining the two hamular notches was used geometrically as the Cartesian x-axis, and the line perpendicular to the x-axis passing through the incisive papilla was defined as the y-axis. The point where both axes meet was defined as the origin of the coordinates.

The midpoint of anterior tooth, mesiobuccal and distobuccal cusp of the correspond posterior tooth was then selected. Let $(O_x, O_y)$ be the coordinate of the origin and $(x_1, y_1)$ be the coordinate of the central incisor. The angle and length of the tooth from the origin (in pixels coordinate) were obtained by $\tan^{-1}\left(\frac{y_1 - O_y}{x_1 - O_x}\right)$ and $\sqrt{(x_1 - O_x)^2 + (y_1 - O_y)^2}$ respectively (Fig. 1). The same procedure was repeated to find the angles and lengths of the other teeth (excluding the third molars) from the origin.

Then, a standard image processing technique was applied where a calibration method converts all measurements in terms of number of pixels into millimeters. The angles $(\theta_i)$ and lengths $(l_i)$ of the teeth from the origin were denoted as $i = 1, 3, \ldots, 17$ for left side of the arch and $i = 2, 4, \ldots, 18$ for right side of the arch. For instance, the length of left central incisor was denoted as $l_1$ and the right central incisor as $l_2$. A combination of angle and length of each tooth from the origin was used collectively to represent the shape of the dental cast [18],[19].

### C. *Dimension Selection*

The dental arch may be assumed to be symmetrical such that only the left side of the arch needs to be considered [20]-[22]. Furthermore, of the 7 teeth on the left side of the arch, only 4 teeth (central incisor, lateral incisor, canine, and distobuccal cusp of first molar) were selected in this study following [23],[24] to represent dental arch shape. Angle and length measurements of these 4 teeth on the left side of the arch were labeled as $\underline{v} = (\theta_1, l_1, \theta_3, l_3, \theta_5, l_5, \theta_{13}, l_{13})$.

### D. *Clustering*

Let $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_{47}$ represent the 47 dental casts studied, and $D(i, j)$ be the Euclidean distance between $\underline{v}_i$ and $\underline{v}_j$ [25]. Agglomerative hierarchical clustering method (Fig. 2) in particular the complete linkage method was applied on the $D(i, j)$ yielding three clusters of $\underline{v}_1, \underline{v}_2, \cdots, \underline{v}_{47}$.

### E. *Principal Component and Box plot*

Let $\overline{\underline{v}}$ be the mean of $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_{47}$. The sample covariance matrix

$$S = \sum_{j=1}^{47} (\underline{v}_i - \overline{\underline{v}})(\underline{v}_i - \overline{\underline{v}})^T$$

was calculated and $\frac{1}{n-1} S$ was used as an estimate of the population covariances, say $\Sigma$. The spectral decomposition of the estimates of $\Sigma$ is given by $Q \Lambda Q^T$ where $Q = (\underline{q}_1 | \underline{q}_2 | \cdots | \underline{q}_8)$ and $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_8)$ are the matrices of eigenvectors and eigenvalues respectively [26].

Henceforth each $\underline{v}_1, \underline{v}_2, \cdots, \underline{v}_{47}$ is represented by its first principal component, namely $\underline{q}_1^T \underline{v}_j \, (j = 1, \ldots, 47)$, and were rearranged in three clusters derived from section D above. Henceforth, box plots of the three groups were plotted (Fig. 3).

## III. RESULTS AND DISCUSSION

Measurements of $(\theta_i, l_i)$ are illustrated in Table I and Table II. Fig. 2 shows the dendrogram using the complete linkage method yielding three clear clusters. Further, the box plot for the first-principal components is given in Fig. 3.

Table I and Table II clearly illustrates that $(\theta_i, l_i)$ were accurately measured using the proposed method, as shown by the small standard deviation values of the measurements.

Clustering of dental arches using $\underline{v}$ followed by the pricipal component were carried out to find the distinct groups of dental arches. It is known that the principal component analysis provides a linear combination of the linear variables. The use of linear variables $(l_i)$ and angular variables $(\theta_i)$ in $\underline{v}$ have minimal effect on the analysis of principal component as the angular variable can be regarded as a linear variable [18],[19].

The dendrogram in Fig. 2 shows three groups. The box plot in Fig. 3 confirms the hypothesis or existence of three groups. Further, an analysis of variance (ANOVA) performed on each group of principal components suggests that there are three distinct groups [26].

TABLE I
MEAN, MEDIAN, STANDARD DEVIATION VALUES, AND RANGE OF ANGLE MEASUREMENTS

| Variable | Mean (SD) (mm) | Min (mm) | Max (mm) | Median (mm) |
|---|---|---|---|---|
| Central incisor ( $\theta_1$ ) | 85.51 (2.14) | 80.68 | 89.39 | 85.25 |
| Lateral incisor ( $\theta_3$ ) | 77.26 (2.32) | 72.33 | 82.47 | 77.03 |
| Canine ( $\theta_5$ ) | 69.62 (2.48) | 64.90 | 64.90 | 69.36 |
| Distobuccal cusp of first molar ( $\theta_{13}$ ) | 38.55 (4.43) | 30.10 | 47.99 | 37.86 |

TABLE II
MEAN, MEDIAN, STANDARD DEVIATION VALUES, AND RANGE OF LENGTH MEASUREMENTS

| Variable | Mean (mm) (SD) | Min (mm) | Max (mm) | Median (mm) |
|---|---|---|---|---|
| Central incisor ( $l_1$ ) | 56.20 (3.33) | 49.20 | 62.66 | 55.76 |
| Lateral incisor ( $l_3$ ) | 54.03 (3.03) | 48.70 | 60.42 | 54.30 |
| Canine ( $l_5$ ) | 50.56 (2.88) | 44.94 | 56.82 | 50.73 |
| Distobuccal cusp of first molar ( $l_{13}$ ) | 35.41 (2.31) | 30.51 | 40.71 | 35.67 |

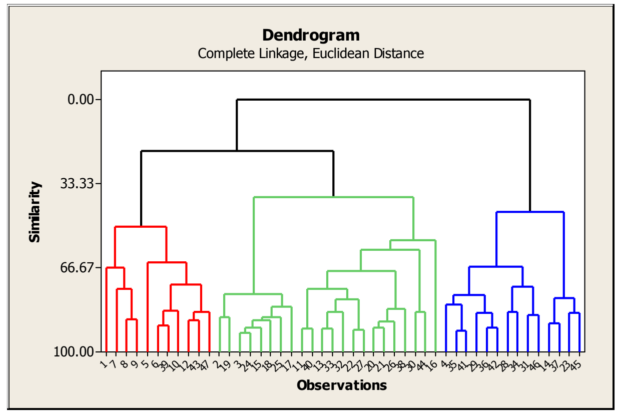


Fig. 2 Three distinct cluster of the dental arch using the complete linkage.

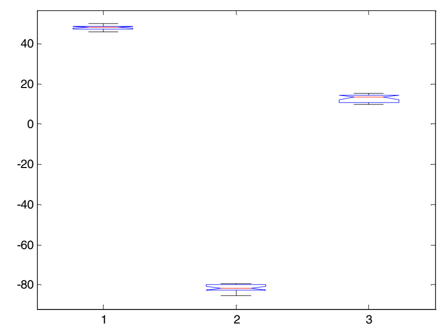


Fig.3. Box plot of the first principal component $\underline{q}_1^T \underline{v}_j \, (j = 1, \ldots, 47)$ .

The existence of three groups of dental arches, namely $\underline{\bar{v}_1} = (83.16, 57.30, 74.85, 54.91, 67.27, 51.49, 36.82, 36.43)$,
$\underline{\bar{v}_2} = (85.55, 53.55, 77.07, 51.61, 69.15, 48.35, 36.11, 33.91)$
and
$\underline{\bar{v}_3} = (87.30, 59.51, 79.43, 57.13, 72.21, 53.30, 43.76, 36.97)$
obtained from this study will help in the design of stock impression trays, in particular for the Malaysian population. Further, this initial information would also help the dental practitioners in making informed choices regarding the selection of impression trays that fit the patients well.

Knowledge of feature $\underline{v}$ and three groups of dental arch also has important applications in other problems in dentistry. In prosthetic dentistry, arrangement of artificial teeth in complete dentures is a skill acquired by dentists through years of clinical observation and experience. An edentulous patient (a person who has lost all his natural teeth) may suffer significantly resorbed (shrunken or lost) alveolar bone (part of the jaw bone that used to support the teeth), especially if the edentulous condition was left untreated for several years [10]. To provide complete dentures to the edentulous patient, the artificial teeth should be placed on the original location that was occupied by the natural teeth (especially the anterior teeth) as this would provide the patient with the optimum aesthetics [11]. The challenge for the restorative dentist therefore is on how to place the teeth on the edentulous arch so that the patient's aesthetics, speech, and mastication functions are restored to the natural pre-extraction condition. Hence, knowledge of feature $\underline{v}$ and three groups of dental arch may provide clinicians with a formal method of determining the positions of the maxillary anterior teeth when rehabilitating edentulous patients.

Another potential area of application is in the field of forensic dentistry. The task of providing dental-related evidence to serve the judicial procedures is relatively new area of study. Analyzing bite marks (associated with murder and rape), identification of dead and dental remains (mass disaster, crime, road accident, drown victim) and human abuse seen in the dental settings (children and elderly) are among the possible application in forensic dentistry [12]. Despite its many successes in solving criminal cases and identifying victims of accidents, the accuracy of their analysis should still be improved. Lack of scientific, empirical or statistical basis (degree of certainty or confidence level) in forensic dentistry, particularly in bite mark analysis has led to wrongfully convictions [13], [14].

Many representations of dental arch shape can be found in the literature, mostly by estimating the arch using a specific or combination of mathematical functions [4]-[9]. In this study, the feature vector $\underline{v}$ not only describes the shape of a dental arch, but also give the knowledge of each tooth location. The results could only be generalized to the Malaysian population, but more data is required to explore the full potential of the proposed ideas.

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