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Robust local estimation in anisotropic diffusion process

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***Abstract*—In this work we propose to use an anisotropic diffusion process using robust statistics. We show that smoothing, while preserving edges, helps the segmentation of upper limb bones (shoulder) in MRI data bases. The anisotropic diffusion equation is mainly controlled using an automatic edge stopping function based on Tukey's biweight function, which depends on the values of gradients pixels. These values are divided into two classes: high gradients for pixels belonging to edges or noisy pixels, low ones otherwise. This process also depends on a threshold gradient parameter which splits both former classes. So a robust local estimation method is proposed to better eliminate the noise in the image while preserving edges. Firstly, the efficiency of the model in the noise reduction is quantified using an entropy criterion on synthetic data with different noise levels to evaluate the smoothing of the regions. Secondly, we use the Pratt's Figure of Merit (FOM) method to evaluate edges preservation. Eventually, a qualitative edge evaluation is given on a MRI volume of the shoulder joint.**

I. INTRODUCTION

Bone structures segmentation is considered as the important tasks for computer visualization and analysis in medical imaging. Particularly the shoulder joint is considered among the most complex because of its many arches of the mobility. For this case, it requires accurate segmentation to obtain a better understanding of its structure and its various movements. Although the reference modality to study bony structures are CT scans, we chose a MR images protocol to study the regions of the shoulder for two reasons. The first is related to the non invasive way of obtaining a data volume with high resolution without exposing the patient to high doses of X-rays. The second reason is due to the possibility of showing both soft tissues and bones of the human body.

However, automatic segmentation of MRI images is difficult for several reasons including the noise which is due to the physical characteristics of acquisition, the intensity variation of pixels within the same tissue, the hypo signal appearance of bony structures, and the partial volume effects due to space discretization.

The manual segmentation is very time consuming, and often causes problems to accurately locate edges in the case of low contrast between two neighboring regions. Thus, an automatic segmentation of MRI data is a real challenge for researchers. The objective is to facilitate the task of the expert while having relevant results for better image interpretation. In this context, the anisotropic diffusion is considered among the most robust process that enables transformation from a noisy image to a smoothed one while preserving edges [1]. Many works focused on the anisotropic diffusion equation. Perona and Malik [2] were the first who studied its characteristics. Then, Catté [3] modified the process using pre-filtered data with the Gaussian kernel filter to regularize data and better calculate the gradient. Later Alvarez [4] introduced a mean curvature term to diffuse the gray level of the image in the direction orthogonal to the gradient. This process is based on a gray level pixels diffusion using an edge stopping function that operates with a threshold on the gradient values of the image.

Several edge stopping functions were used in the literature in order to limit the diffusion across the edges. Thus, the estimation of the gradients corresponding to the edge pixels is needed. The robust estimators give a solution [5] and Black detailed the relationship between the anisotropic diffusion and the robust statistics in [6]. In this context, the edges can be seen as outliers for a robust estimator of the image gradients [7]. This idea is exploited in this paper reconsidering an anisotropic diffusion equation that we previously proposed in [8]. We study the local estimation of the threshold in the Tukey's function in our anisotropic diffusion equation. The next section presents the diffusion model and the stopping function [9]. The third section explains the link between anisotropic diffusion and the robust statistics as well as the interest of this connection in our model.

In the fourth section, we present results on both synthetic and real data. A quantitative evaluation method based on an entropy criterion validates the uniformity in each region and the FOM method [11] validates the edge preservation on synthetic data with different noise levels. A qualitative assessment of contours preservation is shown on a MRI volume of the shoulder joint using image gradients contours.

II. ANISOTROPIC DIFFUSION PROCESS

Anisotropic diffusion as defined by Perona and Malik is a filtering algorithm which removes noise while preserving the contrast at the edges by modifying the image using a partial differential equation. The basic anisotropic diffusion equation is written as follows:

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$$\frac{\partial I_t}{\partial t} = div\left[g\left(\left\|\vec{\nabla} I\right\|\right) \vec{\nabla} I \right] \quad (1)$$

where div is the divergence operator, $\vec{\nabla}$ is the nabla operator and g is the stopping function that controls the diffusion process.

Jerbi et al. [8] formulates this anisotropic diffusion equation (1) as a fronts propagation based on level sets. Diffusion is thus seen as an evolution of several surfaces of the same level under the influence of different propagation speeds. This connection between the front propagation and the anisotropic diffusion taking into account the improvements introduced by Catté and Alvarez, gave the basic diffusion equation:

$$\begin{cases} I_{t=0} = I_0 \\ \dfrac{\partial I_t}{\partial t} = -\underbrace{\alpha\, H(I_t)\, g\left(\left|\vec{\nabla}\tilde{I}_t\right|, \gamma\right)\left|\vec{\nabla} I_t\right|}_{(1)} + \underbrace{\beta\, \vec{\nabla} g\left(\left|\vec{\nabla}\tilde{I}_t\right|, \gamma\right).\vec{\nabla} I_t}_{(2)} \\ \qquad - \underbrace{\gamma\left(I_t - I^*_{t-\varphi}\right)\left|\vec{\nabla} I_t\right|}_{(3)} \end{cases} \quad (2)$$

where t denotes the time scale parameter, H is the mean curvature of the image I_t and $\tilde{I}_t$ is the spatial smoothed data with a Gaussian kernel.

The first term of equation (2) is the diffusion term that generates nonlinear front propagation and ensures edge preservation using the g function. A detailed analysis of this function will be explained later in the paper.

The second term is useful for the edge contrast enhancement.

The third term represents a data fidelity term that allows the control of the diffusion process from the already filtered data I^* between $(t-\varphi)$ and t, φ being a time delay. This term was accurately studied in [8].

We note that α, β, and γ are the parameters corresponding to the three speed terms of the anisotropic diffusion equation and are manually fixed. The focus of this paper is therefore the first speed term which evaluates the influence of the edge stopping function on the diffusion. Thus β and γ were set to 0.

In the literature, several stopping functions are chosen to satisfy the two following conditions:

i) $g(x,\gamma) \to 0$: when the amplitude of x gradients is high, the diffusion is stopped at the edges, ii) $g(x,\gamma) \to 1$: when the amplitude of x gradients is low, the diffusion reaches its maximum. These conditions provide both image smoothing and edge preservation. The scale γ parameter represents the threshold at which the diffusion is stopped. A complete study of this parameter will be given below.

Several studies have focused on the stopping function that characterizes the process of anisotropic diffusion. Perona and Malik [2] proposed two functions: "Leclerc" and "Lorentz". Subsequently, Black [6] proposed to use the Tukey's stopping function which enables to completely stop the diffusion process for pixels whose the gradient value exceeds the reject point γ, while for both other functions, the diffusion continues to smooth data.

According [9], we are adopting the form of Tukey's biweight function in our model for the g function:

$$g(x,\gamma) = \begin{cases} \left(1 - \left(\dfrac{x}{\gamma}\right)^2\right)^2 & , \ x \le \gamma \\ 0 & otherwise \end{cases} \quad (3)$$

As explained above, this function depends on the parameter γ that needs to be estimated. This parameter is used to ensure the success of the diffusion process reducing the noise as much as possible and preserving edges. In the next section, we explain the method based on robust statistics to determine this threshold.

III. ROBUST ESTIMATION AND ANISOTROPIC DIFFUSION PROCESS

In the literature, several techniques of robust statistics have been applied in the field of computer vision. These methods tend to modify a cost function in order to limit outliers' influence. The main consequence is to limit the speed of convergence in the optimization algorithms.

In this context Black [6] proposes to establish a link between robust statistics and anisotropic diffusion. Indeed, anisotropic diffusion can be seen as a problem of estimating a piecewise constant image from a noisy image. In this case the segmentation problem is defined as a minimization of a robust norm of the difference between a gray level image pixel and its neighbors. We note $\rho(\cdot)$ the robust norm which is the cost function to minimize. This connection is given by $g(x,\gamma).x = \rho'(x,\gamma) = \psi(x,\gamma)$, where γ denotes the threshold gradient and ψ the influence function, characterizing the effect of outliers i.e. in our case, the edge gradients of the image.

This function reaches its maximum for a threshold γ and decreases after in order to reach its null value for another threshold $\gamma_e = \sqrt{5}\gamma$. This link allows us to deduce robust methods for estimating the threshold parameter. For example, based on [5], the method of M-estimators using the MAD operator (Median Absolute Deviation) was used successfully in several applications [6, 8]. We choose $\gamma = 1.4826 MAD(|\bullet|)$.

This method gives very good results if the proportions of low and high gradient are homogenously distributed along the image. It means that there are as many region pixels as edge pixels but this is rarely the case. This led us to propose a local estimation method for calculating the MAD from a portion of the image and not from the entire image to respect the assumptions of the estimator. To apply this method in our iterative scheme, the threshold parameter γ must be initialized by calculating it on the entire image. After each iteration i, γ_i is updated and estimated from the pixels selected by γ_{i-1}. This method minimizes the effect of the gradients majority in the images (high gradients or low gradients).

This new local and robust method, estimating the threshold gradient associated with the edge stopping function based on Tukey's biweight allowed us to ameliorate results in comparison to the use of the global estimation [9]. A better noise removal and edge preservation are obtained as shown in the following section.

IV. Evaluation and Results

To evaluate our result we propose to use synthetic data building from the manual segmentation of MRI slices corresponding to the shoulder joint. We obtain a volume of 172x255x146 voxels with a resolution of 0.7x0.7x1mm^3 composed of four homogeneous areas of a gray level (60, 95, 157, 176). Thus we have three regions and one edge. Finally we add a Gaussian kernel with different noise levels (σ=5, σ=10, σ=20) as shown in Figure 1, middle column.

Figure 1 shows the results of the anisotropic diffusion equation applied on synthetic data using both global and local estimation during 50 iterations. We note that the three areas-regions become very homogeneous while preserving precisely the zone-edge. Also we show that the global estimation with the Tukey's stopping function (on the left column) not completely eliminates noise and the boundary between regions is altered. However using local estimation with the same edge stopping function, the results (on the right column) are significantly improved by eliminating almost the noise while preserving edges. These anisotropic diffusion results are performed using two different quantitative measures to validate the smoothed synthetic data.

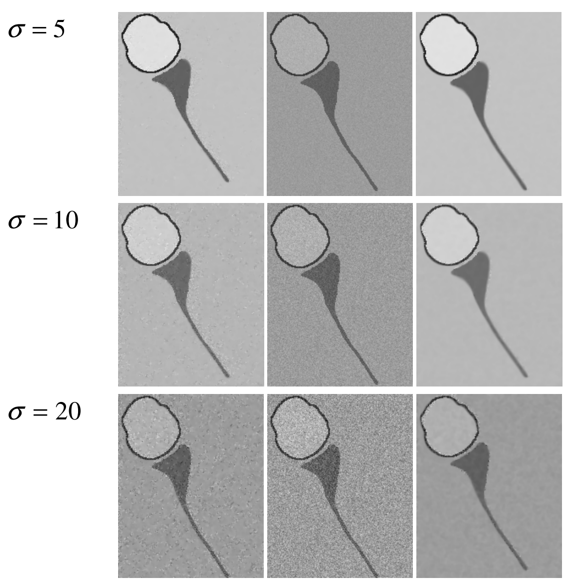

Fig 1: anisotropic diffusion results (in the middle initial slices, on the right smoothed results using Tukey's function with local estimation, on the left smoothed results using Tukey's function with global estimation).

A. *Evaluation of the region segmentation*

To validate the smoothing degree obtained by our model, we use an entropy criterion [8, 10] which evaluates the homogeneousness into the regions during the iterations. The results of the evaluation on synthetic data with different noise levels are presented in Figure 2.

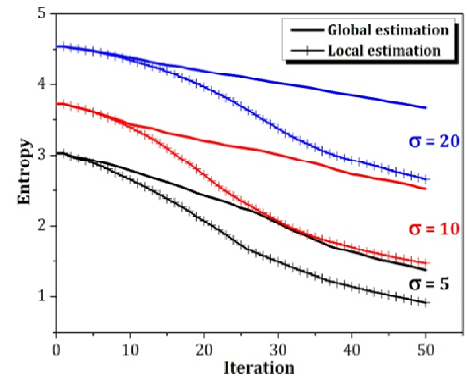

Fig 2: entropy curves of anisotropic diffusion results on synthetic data using local and global estimation.

For each level noise, we note that the proposed model with the local estimation reaches lower entropy compared to the model with the global estimation.

B. *Evaluation of the edges segmentation*

To quantify the result of anisotropic diffusion about the edges location, we use the FOM (Pratt's Figure of Merit) [11]. This method is used in several works which study the edge detection process. It requires the edges of reference here obtained by morphological operators applied on synthetic data without noise. Then we use the following expression to calculate the FOM:

$$FOM = \frac{1}{\max\left(N_{ref}, N_{result}\right)} \sum_{i=1}^{N_{result}} \frac{1}{1+\alpha d_i^2} \quad (3)$$

where N_{ref} is the number of edge pixels of the synthetic reference data, N_{result} is the number of edge pixels in the gradients image result, d_i denotes the distance between the i^{th} current edge and the corresponding detected edge and α is a scaling constant fixed to 1/9 as in Pratt's works [11].
The FOM values must range between 0 and 1 (best results).
To correctly apply this evaluation method, we must choose a threshold distance. This condition enables to avoid the aberrant distances due to aberrant gradients located on the image border. In our case we set the threshold value to 10.
Then, we determine the FOM value for each slice of the resulting synthetic volume obtained by applying the local and global estimation in the diffusion process (Fig. 3).

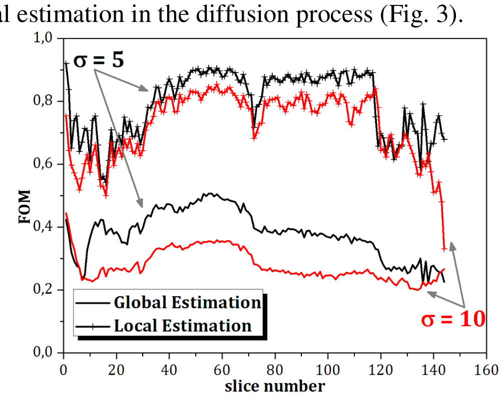

Fig 3: curves of FOM value for each slice of the resulting synthetic volume.

We note that using our proposed model (Tukey's function with local estimation) FOM values are higher than using a model with global estimation for both noise levels (σ=5, σ=10). The local model better locates the edges.
After the quantified validations of the method on synthetic data, we apply it on real MRI data of the shoulder.
Figures 4 and 5 show the resulting volume processed by the robust anisotropic model using a local estimation (Fig. 5) compared to the previous model using a global estimation (Fig. 4). In Figure 5, the regions are more homogeneous than in Figure 4 and the edges are thinner and well located around the surfaces of the main bone structures.

V. CONCLUSION AND FUTURE WORK

In this paper, we propose an anisotropic diffusion process using the Tukey's edge stopping function based on a local robust estimation using a MAD operator. This proposed model enables us to better eliminate noise while preserving edges.
To evaluate the model performance and to compare the results, we use both quantitative evaluation of regions (entropy) and edges (Pratt's Figure of Merit) on synthetic data whose the ground truth is known. We also used a qualitative evaluation on MRI data.
In further works, we will focus on the activation of the second term of the diffusion equation to show its influence on the improvement of the image contrast and the edges continuity. Thus, we should obtain an accurate segmentation helpful to better understand the shoulder's structures and its various movements. We will apply our method to other imaging modalities such as CT images for quantifying the bone necrosis degree.

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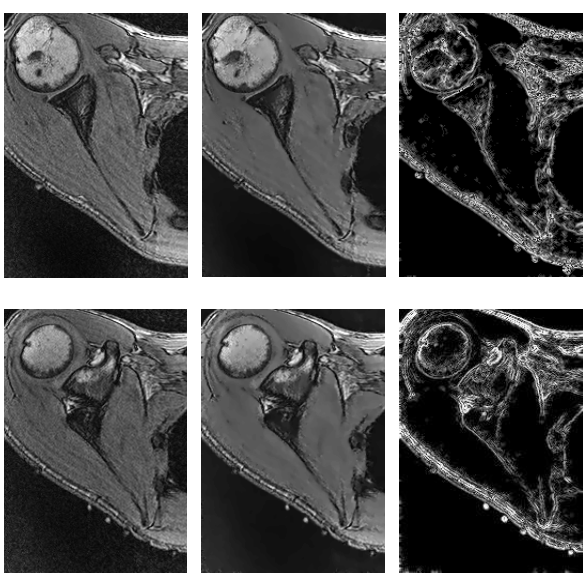

Fig 4: Anisotropic diffusion process on MRI of the shoulder: 96th and 108th initial slice (left), diffused results using robust **global** estimation (middle), the corresponding edge pixels results (right).

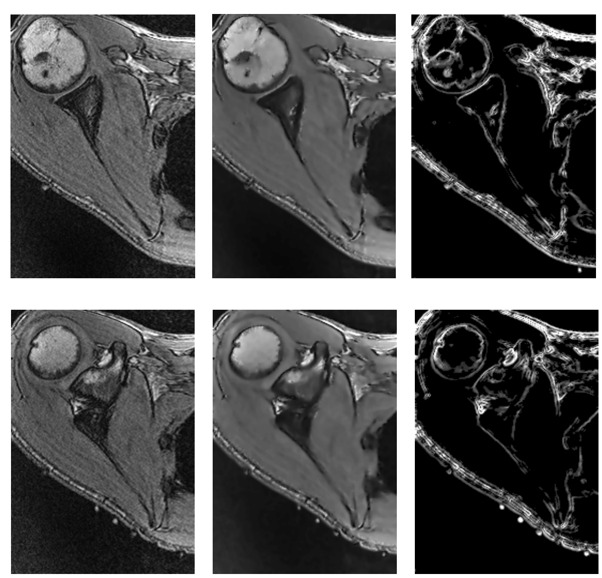

Fig 5: Anisotropic diffusion process on MRI of the shoulder: 96th and 108th initial slice (left), diffused results using robust **local** estimation (middle), the corresponding edge pixels results (right).