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Reference-Guided Sparsifying Transform Design for Compressive Sensing MRI

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***Abstract*— Compressive sensing (CS) MRI aims to accurately reconstruct images from undersampled k-space data. Most CS methods employ analytical sparsifying transforms such as total-variation and wavelets to model the unknown image and constrain the solution space during reconstruction. Recently, nonparametric dictionary-based methods for CS-MRI reconstruction have shown significant improvements over the classical methods. These existing techniques focus on learning the representation basis for the unknown image for a synthesis-based reconstruction. In this paper, we present a new framework for analysis-based reconstruction, where the sparsifying transform is learnt from a reference image to capture the anatomical structure of unknown image, and is used to guide the reconstruction process. We demonstrate with experimental data the high performance of the proposed approach over traditional methods.**

I. INTRODUCTION

The emerging theory of compressive sensing (CS) has created significant interest in magnetic resonance imaging (MRI), as it offers a robust and systematic framework for increasing the imaging speed which is inherently low in MRI. This slow acquisition is prohibitive in many important clinical applications, especially in dynamic and interventional imaging, where near real-time acquisition speeds are desired. Despite the advances in imaging hardware, the acquisition speed is limited by MR physics and prohibitive in many applications.

Compressive sensing provides a systematic methodology to recover images from a significantly smaller number of measurements than that required by Nyquist sampling. The underlying principle to make this possible is the sparsity of the images in some transform domain. An extensive body of work investigated a number of different sparsifying transforms that exploit the image characteristics for accurate reconstruction. Classical approaches consider non-adaptive, analytical transforms such as wavelets, total-variation, and contourlets [1], [2]. More recently, adaptive transforms (or dictionaries) have become increasingly popular as they can more accurately capture characteristics of the images of interest in a particular application. Early work in this area focused on analytical derivation of parametric bases [3], whereas more recently, nonparametric dictionary learning has been developed and applied to many problems in imaging [4]–[7]. The work in [8] proposed reference-guided weights to analytical transforms during reconstruction.

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Most of the work in dictionary-based recovery utilizes learning a dictionary from a collection of precollected reference images. The trained dictionary is then used as the sparse representation basis for the image, and which is then employed in a synthesis-based reconstruction algorithm typically minimizing the l_p-norm ($0 \leq p \leq 1$) of the representation coefficients. The work [9] also proposed joint dictionary learning and sparse reconstruction which showed significant improvements over existing methods.

The main principle behind these methods is to constrain the characteristics of the reconstructed image to certain anatomical structures, rather than blindly and uniformly enforcing local smoothness as in traditional methods. The morphology of the subject of interest is generally known to an high extent, which provides valuable information to guide the reconstruction process. In this paper, we propose a novel algorithm based on the same principle. However, rather than constraining the representation basis, an adaptive sparsifying transform is learnt from a reference image and used in an analysis-based reconstruction. This transform is essentially spatially-varying high-pass filters which enforce local anatomical structure (learnt from a reference image) during reconstruction. We demonstrate with experimental results that the proposed method is very effective for compressive sensing MRI reconstruction while maintaining robustness.

The rest of this paper is organized as follows. The proposed formulation is described in Section II. Experimental validation of the approach is done in Section III, followed by concluding remarks in Section IV.

II. PROPOSED FORMULATION

The goal in compressive sensing MRI is to reconstruct the unknown image $\boldsymbol{\rho} \in \mathbb{C}^p$ from limited k-space measurements $\mathbf{d} \in \mathbb{C}^m$ with $m \ll p$. The imaging system is commonly modeled as

$$\mathbf{d} = \mathbf{F}_u \boldsymbol{\rho} + \mathbf{n}, \tag{1}$$

where $\mathbf{F}_u \in \mathbb{C}^{mxp}$ is the undersampled Fourier encoding matrix, and $\mathbf{n}$ is the observation noise. Typical formulations of the CS reconstruction problem use either the analysis-based regularization as [10]

$$\hat{\boldsymbol{\rho}} = \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \parallel \mathbf{d} - \mathbf{F}_u \boldsymbol{\rho} \parallel_2^2 + \lambda\, R\left(\boldsymbol{\Psi}\boldsymbol{\rho}\right), \tag{2}$$

or, synthesis-based sparse reconstruction using

$$\hat{\boldsymbol{\alpha}} = \underset{\boldsymbol{\alpha}}{\operatorname{argmin}} \parallel \mathbf{d} - \mathbf{F}_u \boldsymbol{\Phi}\boldsymbol{\alpha} \parallel_2^2 + \lambda\, R\left(\boldsymbol{\alpha}\right), \tag{3}$$

and setting $\hat{\boldsymbol{\rho}} = \boldsymbol{\Phi}\hat{\boldsymbol{\alpha}}$. The parameter λ controls the strength of the regularization. In both cases, the regularization functional $R(\cdot)$ is generally a variant of the l_p-norm with $0 \leq p \leq 1$, which is known to enforce sparsity in the solutions. Although the difference between the two approaches might appear to be subtle, they deviate significantly both in terms of modeling and algorithm development [10]. The analysis-based algorithms are common in many inverse problems; algorithms based on total-variation, and x-let (wavelet, curvelet, etc.) *forward* (or *analysing*) transforms for Ψ, fall into this category. The synthesis-based approach is relatively more recent, and it is based on the assumption that the signal $\boldsymbol{\rho}$ can be represented as a linear combination of columns of $\boldsymbol{\Phi}$, and this representation $\boldsymbol{\alpha}$ is sparse.

In both approaches, the transforms $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are chosen traditionally as *data-independent*, i.e., analytical transforms with certain desirable properties such as fast evaluation and inversion. The development of data-dependent, nonparametric dictionaries showed considerable improvement over analytical ones due to better and more specific modeling of image classes [3]–[7]. As the synthesis-based approach is constructive, and therefore more intuitive for modeling, all dictionary learning approaches are developed using this approach. To our knowledge, dictionary learning based on the analysis approach is not investigated, although analysis-based optimization is easier to develop and solve. Our proposal in this work is to develop such an approach for reference-guided reconstruction of MR images from undersampled k-space measurements.

The most important property in designing the analysing transform $\boldsymbol{\Psi}$ is its sparsifying ability, i.e., a good transform should produce highly sparse coefficients such that imposing $R(\boldsymbol{\Psi\rho})$ significantly constrains the solution space of $\boldsymbol{\rho}$. The transform $\boldsymbol{\Psi}$ should accurately capture the local spatial activity of image $\boldsymbol{\rho}$, and in the ideal case, its application to the image will result in uncorrelated noise.

Analytical analysis operators such as total variation and wavelets impose sparseness on all preselected filters. For instance, total-variation suppresses finite differences in both horizontal and vertical directions in the whole image domain. However, if prior knowledge on the edge structure is available, the suppression of image variation can employ this information and follow the edge structure, which in turn will result in higher sparsity levels.

In most MRI applications, a reference fully-sampled image $\boldsymbol{\rho}^{\mathrm{ref}}$ can be obtained to extract the local anatomical structure within the image. This information can be effectively used to adaptively design filters to impose the learnt local anatomical structure during the reconstruction process.

Based on this, we define the analysis operator as spatially-varying high-pass filters defined at each pixel location r:[1]

$$\Psi(r)=\delta(r)-\frac{1}{K_r}\sum_{\substack{s\in\mathcal{N}(r)\\ s\neq r}}\delta(s)f\left(r,s\right)g\left(\rho_r^{\mathrm{ref}},\rho_s^{\mathrm{ref}}\right), \quad (4)$$

[1]Note that the r^{th} row of the operator $\boldsymbol{\Psi}$ defines a filter for a particular location r.

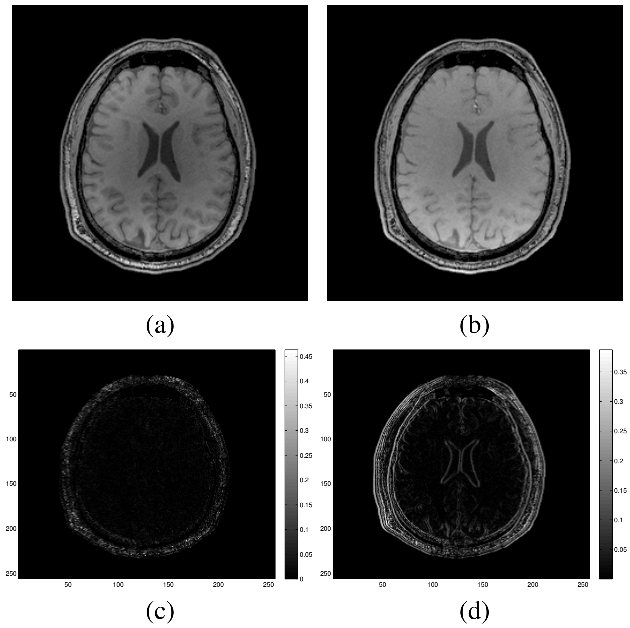

Fig. 1. (a) Original image, and (b) reference image taken from a diffusion MR experiment. Both images are normalized to unit maximum intensity. The magnitudes of the coefficients of the output of the filters learnt from the reference image are shown in (c), and the magnitudes of horizontal gradients in the original image are shown in (d).

where $\delta(r)$ is the delta function, $\mathcal{N}(r)$ defines a rectangular neighborhood around pixel r, and K_r is the normalization factor. The filter $\Psi(r)$ consists of two parts: A spatially invariant filter $f(r,s)$, used to weigh closer pixels higher than pixels far apart; and a spatially-varying filter $g\left(\rho_r^{\mathrm{ref}},\rho_s^{\mathrm{ref}}\right)$, computed using the local spatial variations in the reference image. Conceptually, $g\left(\rho_r^{\mathrm{ref}},\rho_s^{\mathrm{ref}}\right)$ must assign larger weights to pixels with similar intensity values, such that the filtering is performed along the edges, and filtering across edges is suppressed. Based on this, we define $g(\cdot)$ to be a function of the radiometric distance between pixels r and s, such that $g\left(\rho_r^{\mathrm{ref}},\rho_s^{\mathrm{ref}}\right) = g\left(|\rho_r^{\mathrm{ref}}-\rho_s^{\mathrm{ref}}|\right)$. Any monotonically decreasing function can be used for $g(\cdot)$; in this work, we use the absolute differences such that

$$g\left(\rho_r^{\mathrm{ref}},\rho_s^{\mathrm{ref}}\right) = \frac{1}{|\rho_r^{\mathrm{ref}}-\rho_s^{\mathrm{ref}}+\epsilon|}, \quad (5)$$

where ϵ is a small number which accounts for the observation noise in the reference image, and for possible anatomical discrepancies between the reference and unknown images.

As an example of how much structural information can be obtained using the proposed analysis operator, consider the image pair in Fig. 1, extracted from a dynamic MR sequence. We have constructed the filter $\boldsymbol{\Psi}$ using the reference shown in Fig. 1(b) and applied it to the image in Fig. 1(a). The reference and original images have different local and global contrast, and they have different structure around the sculp. The magnitudes of the coefficients at the filter output is shown in Fig. 1(c). For comparison, the absolute values of the horizontal differences are shown in Fig. 1(d). Notice that the proposed filter produces a much sparser output, and captures the characteristics of the image to a greater extent.

In addition, the filtered image have noise-like characteristics, and hence much of the anatomical structure is captured by the proposed filter, whereas the horizontal gradients exhibit significant image structure, as expected.

Using the reference-guided analysis transform described above, the reconstruction of the image $\boldsymbol{\rho}$ can be performed using the classical optimization shown in (2). Employing the standard l_1-minimization formulation, (2) becomes

$$\hat{\boldsymbol{\rho}} = \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \parallel \mathbf{d} - \mathbf{F}_u \boldsymbol{\rho} \parallel_2^2 + \lambda \parallel \boldsymbol{\Psi} \boldsymbol{\rho} \parallel_1 , \tag{6}$$

which can be solved using a number of existing methods. Here we use the reweighted least squares method as a reference method to compare the proposed design with traditional algorithms.

III. RESULTS AND DISCUSSION

A. Simulation Results

The proposed method has been evaluated systematically using different reference images with varying degrees of similarity to the original image. To demonstrate the advantage of the method, we include comparisons with two methods; the traditional TV-based CS reconstruction [1], and an l_1-based algorithm that solves

$$\hat{\boldsymbol{\rho}} = \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \parallel \mathbf{d} - \mathbf{F}_u \boldsymbol{\rho} \parallel_2^2 + \lambda \parallel \boldsymbol{\rho} - \boldsymbol{\rho}^{\text{ref}} \parallel_1 . \tag{7}$$

This method uses the reference image directly in the reconstruction as an additional constraint. In all algorithms, the algorithmic parameters are optimized in each case as to represent their best reconstruction results.

The unknown target image is chosen from a time series dynamic MRI experiment, and is shown in Fig. 1(a). A variable density sampling pattern is used to acquire undersampled k-space data at various undersampling ratios. We test the proposed approach with three different reference images: the target image, which is used to provide an "oracle" performance bound; the target image corrupted with additive Gaussian noise of variance 0.01; and another frame from the same dynamic MR sequence, shown in Fig. 1(b). The noisy target image as the reference is used to assess the robustness of the proposed method to noise in the reference. As mentioned above, the reference image in Fig. 1(b) contains the general anatomical structure, but it has different local intensity variations and observation noise. Therefore, it represents a realistic reference image that can be acquired in many applications.

As the quantitative quality measure we use the relative error, defined by $\frac{\|\hat{\boldsymbol{\rho}}-\boldsymbol{\rho}\|_2}{\|\boldsymbol{\rho}\|_2}$, where $\boldsymbol{\rho}$ is the original image, and $\hat{\boldsymbol{\rho}}$ is the reconstructed image, respectively. Figure 2 shows the performance of the algorithms with different reference images and different undersampling ratios. The proposed algorithm is denoted with preceding P-, and the formulation in (7) is denoted with preceding l_1-, with different reference images. Reconstructed images at undersampling factors of 15% and 25% are shown in Fig. 3 and Fig. 4, respectively.

Several important remarks can be made from the results. First, the proposed approach provides better reconstruction performance at all undersampling factors and with all reference images. Second, while the performance of the approach degrades when reference images with lower similarity are used, this degradation is not severe (around 2-3$\%$), and the method provides performance close to the oracle bound. It can be stated that if the reference image contains the general anatomical structure (albeit degraded), the proposed approach is able to exploit this information to an high extent during reconstruction. On the contrary, the l_1-based reconstruction using the reference image is highly prone to the noise in the reference image, and the relative reconstruction error is mostly determined by the noise in the reference image. In addition, when an image with different contrast characteristics is used, its performance decreases significantly and becomes close to the TV reconstruction. Finally, the TV-reconstruction is not able to provide accurate reconstructions; the reconstructed images either exhibit large aliasing artifacts or oversmoothing.

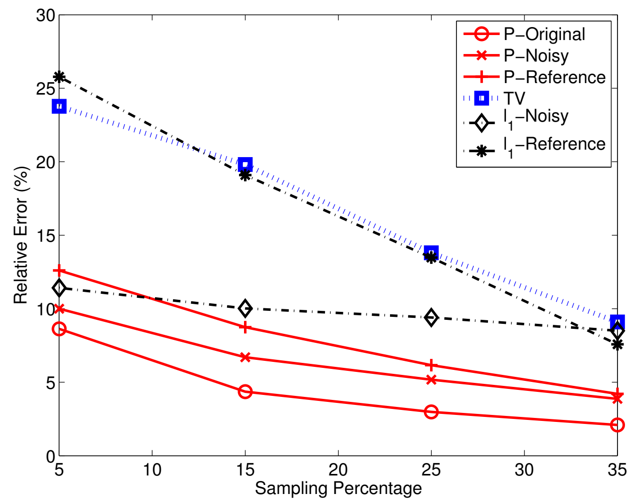

Fig. 2. Relative reconstruction error vs. sampling percentage for all algorithms with different reference images.

B. Discussion

The proposed method can be employed for a variety of imaging applications where reference images can be collected that contain the anatomical structure. Potential such applications include interventional MRI, diffusion-weighted MRI, and MR spectroscopy imaging. To more effectively utilize the proposed approach, a variety of experiments (e.g., with different contrast weightings or diffusion tensor imaging) can be performed to acquire additional information for the design of the analysis transform (see a related discussion in [8]).

In cases where the expected similarity between the target and reference images can be estimated, this knowledge can be incorporated in the transform design using the filter parameters (such as ϵ in (5)). The parameter can be adjusted to obtain more robustness to discrepancies between the target and reference images. In addition, similar to the method in [9], the filters learnt from the reference image can be adapted using the reconstructed images during the iterative procedure.

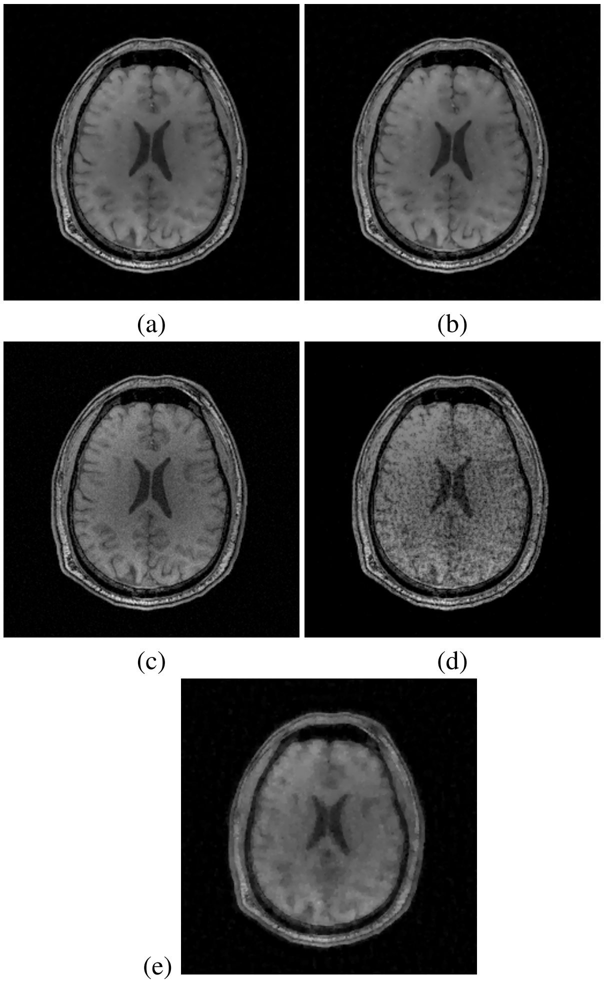

Fig. 3. Reconstructed images at 15% sampling ratio. *Top row:* Proposed method, *middle row:* l_1-based method using (7), *bottom row:* TV reconstruction. (a,c) is obtained with the noisy target image as the reference, and (b,d) is obtained with the image in Fig. 1(b) as the reference.

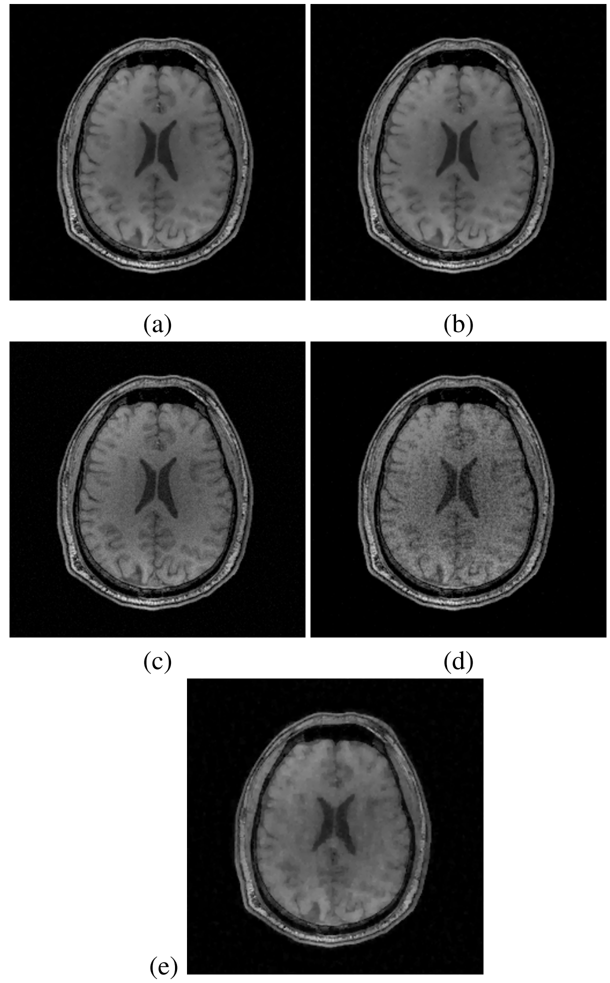

Fig. 4. Reconstructed images at 25% sampling ratio. *Top row:* Proposed method, *middle row:* l_1-based method using (7), *bottom row:* TV reconstruction. (a,c) is obtained with the noisy target image as the reference, and (b,d) is obtained with the image in Fig. 1(b) as the reference.

These approaches will be considered in future work. Finally, although the focus of this work is compressive sensing reconstruction, the proposed construction of the analysis operator can also be used for other recovery applications, such as denoising and motion artifact removal.

IV. CONCLUSIONS

We presented a novel method for compressive sensing MRI based on the design of reference-guided analysis transforms. We have demonstrated that this formulation achieves a high degree of sparsity and is able to capture and enforce the salient anatomical structure during the reconstruction process. The proposed method has shown to provide higher reconstruction performance compared to methods using analytical transforms and to methods using the reference images directly as constraints.

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