# Latency variable source separation for heart rate detection in low-quality ECG signals

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Abstract-Monitoring of the heart rate can provide vital clinical information, but can, in specific situations, be complicated due to the low signal to noise ratio (SNR) of the available physiological signals. Several methods to enhance the SNR are known from literature, e.g. wavelet-based enhancement methods, but most of these methods require a priori information on the recorded signals and are only applicable in specific situations. In this paper a generic method is presented that uses latency variable source separation (LVSS) to derive a matched filter for enhancement of electrocardiogram (ECG) signals. Besides its use on ECG signals, the LVSS method has the potential capability to enhance any kind of (quasi-) periodical signal. The LVSS method is evaluated by comparing its performance in SNR enhancement to the performance of a wavelet-based enhancement method. This performance demonstrates that for low-SNR ECG signals, the LVSS method outperforms the wavelet-based method.

### I. INTRODUCTION

The heart rate provides relevant clinical information in many cases, ranging from monitoring of the fetus during pregnancy to monitoring of athletes during exercise. For both of the indicated applications, different methods exist for obtaining the physiological signals that are needed to assess the heart rate. For instance, the fetal heart rate is generally monitored using Doppler ultrasound, while the heart rate of athletes is often monitored using electrocardiography. Although these methods are very different, they have the common aspect that in some specific situations the signal to noise ratio (SNR) of the acquired signals is relatively low, complicating assessment of the heart rate.

To assess the heart rate from signals with relatively low SNR, several methods have been proposed in literature [1], [2], [3], [4]. Most of these methods operate by a priori enhancing the SNR of the recorded physiological signals, e.g. through wavelet analysis [2], [3] or bandpass filtering [4], and subsequent peak detection by means of autocorrelation techniques or (adaptive) thresholds [5]. The main drawback of these methods is that their performance strongly depends on the success of the preprocessing techniques. Since most of the preprocessing techniques are developed for specific kinds of physiological signals (e.g. electrocardiographic or ultrasonic signals) or specific SNR levels, their performance deteriorates when, for instance, the SNR of the recorded signals is significantly lower than anticipated or when the recorded signals have a different appearance/waveform than expected. Hence, the need exists for a robust preprocessing

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technique – or more generally, for a heart rate detection method – that performs sufficiently well on physiological signals of all kinds and qualities.

In this paper, we develop a preprocessing technique that enables matched filtering of the acquired physiological signals. For signals that are corrupted by additive stochastic noise, a matched filter is the linear filter that maximizes the SNR and as such constitutes an optimal preprocessing technique. During the matched filtering, the recorded physiological signals are convolved with a template that, in theory, matches the (quasi-) periodical waveform contained in the recorded signals. For electrocardiographic signals, this periodic waveform would be the electrocardiogram (ECG). The main reason why matched filtering has, up till now, not been applied successfully for heart rate detection is that it is rather impossible to generate an accurate template for the periodic waveform without prior knowledge on the heart rate [6]. To overcome this problem, attempts have been made to use more generic templates for the matched filter [6], [7], however leading to less accurate matching and, as a result, to less SNR enhancement. In this paper, we overcome this problem by using latency variable source separation (LVSS) to build a template for the periodic waveform in the recorded signals and use this template in a matched filter.

In the LVSS, the physiological signals are divided into short segments. Each of these segments is assumed to represent a linear combination of several underlying sources, each of them scaled independently and each of them with a different latency. By using a Bayesian approach, the unknown linear combination, the scaling, and the latency can be estimated and corrected for. By subsequently averaging the corrected segments, a template for the periodic waveform can be generated. This template is thus generated without using any prior information on the heart rate. Moreover, based on the (fixed) length of the segments and the estimated latencies, an initial estimate for the heart rate can already be determined.

In this paper, we focus on the determination of the heart rate from ECG recordings, but, as mentioned before, the described methodology can also be applied on other (quasi-) periodical signals.

# II. SEPARATION OF SOURCES WITH VARIABLE LATENCY AND AMPLITUDE

### A. Probabilistic ECG model

When M ECG signals are simultaneously recorded, and when the recorded ECG signals are divided in R individual segments of fixed length T, then each epoch r of ECG

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Fig. 1. Block diagram of the ECG segment model. The model is described for the  $r^{\text{th}}$  segment/epoch.

segments can be described by the  $[M \times T]$  matrix  $\mathbf{V}_r$ . These ECG segments are assumed to originate from the linear combination  $\mathbf{C}$  ( $[M \times N]$ ) of N independent sources  $\mathbf{S}$  ( $[N \times T]$ ), where for each epoch, the sources can have an individual latency  $\psi_{nr}$  and scaling  $\alpha_{nr}$ . Finally, the ECG segments are assumed to be corrupted by the  $[M \times T]$  additive noise matrix  $\eta$ . This is illustrated in Fig. 1 and described mathematically as:

$$V_{mr}(t) = \sum_{n=1}^{N} C_{mn} \alpha_{nr} S_n(t - \psi_{nr}) + \eta_{mr}(t), \qquad (1)$$

with  $r = 1 \dots R$  and  $m = 1 \dots M$ .

The independent sources **S** can, next to ECG signals, represent interferences originating from e.g. muscle activity or noise from non-physiological sources. Any interference or noise in the recorded signals that is not included in any of the sources is assumed to be modeled by the noise  $\eta$ . Since, in a first-order approximation, the ECG can be considered as the linear combination of three independent sources [8], it is rather straightforward to assume that, in case of multichannel  $(M \ge 1)$  ECG recordings, three of the sources **S** represent an ECG signal. Because these ECG sources have to be synchronized, their latencies  $\psi_{nr}$  have to be the same:

$$\psi_{nr} = \phi_r$$
 for  $n = 1, 2, 3.$  (2)

When following a probabilistic approach, the probability distribution for the parameters of interest, i.e. *C*, **S**,  $\alpha$ ,  $\psi$ , and  $\phi$ , given the recorded ECG segments can be described as  $p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V})$  [9]. Using Bayes' rule, this probability distribution can be written as:

$$p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V}) = \frac{p(\mathbf{V} | C, \mathbf{S}, \alpha, \psi, \phi) p(C, \mathbf{S}, \alpha, \psi, \phi)}{p(\mathbf{V})}.$$
(3)

By assuming statistical independency and uniform distributions for p(C),  $p(\alpha)$ ,  $p(\mathbf{S})$ ,  $p(\psi)$ , and  $p(\phi)$ , assuming  $\eta$  to be a zero-mean Gaussian noise signal with variance  $\sigma^2$ , and the probability distribution for this variance to be described by a Jeffreys prior (i.e.  $p(\sigma) \propto \sigma^{-1}$ ) [9], Eq. (3) can be simplified to:

$$p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V}) = Q^{-\frac{MRT}{2}} \Gamma\left(\frac{MRT}{2}\right), \qquad (4)$$

with

$$Q = \sum_{m=1}^{M} \sum_{r=1}^{R} \sum_{t=1}^{T} \left[ V_{mr}(t) - \sum_{n=1}^{N} C_{mn} \alpha_{nr} S_n(t - \psi_{nr}) \right]^2.$$
 (5)

### B. Inference on model parameters

The most probable set of model parameters  $\hat{C}$ ,  $\hat{S}$ ,  $\hat{\alpha}$ ,  $\hat{\psi}$ , and  $\hat{\phi}$  can be estimated by setting the first derivative of the log posterior, i.e.  $\ln p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V})$ , with respect to any of the parameters equal to zero while keeping the other parameters fixed. For the inference of the sources  $\hat{S}$  this derivative is:

$$\frac{\partial \ln p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V})}{\partial S_j(\tau)} = -\frac{MRT}{2}Q^{-1}\frac{\partial Q}{\partial S_j(\tau)}.$$
 (6)

Hence, the optimal source  $\hat{\mathbf{S}}$  can be inferred by setting the derivative of Q with respect to  $S_j(\tau)$  equal to zero, resulting in:

$$\hat{S}_{j}(\tau) = \frac{1}{\sum_{m} \sum_{r} C_{mj}^{2} \alpha_{jr}^{2}} \begin{cases} \sum_{m} \sum_{r} Z_{\text{ECG}} C_{mj} \alpha_{jr}, & j \leq 3 \\ \sum_{m} \sum_{r} Z_{\text{other}} C_{mj} \alpha_{jr}, & j > 4 \end{cases}$$
(7)

with

$$Z_{\text{ECG}} = V_{mr} \left(\phi_r + \tau\right) - \sum_{\substack{n=1\\n\neq j}}^{3} C_{mn} \alpha_{nr} S_n \left(\tau\right)$$
$$- \sum_{n=4}^{N} C_{mn} \alpha_{nr} S_n \left(\phi_r - \psi_{nr} + \tau\right)$$
(8)

and

$$Z_{\text{other}} = V_{mr} \left( \psi_{jr} + \tau \right) - \sum_{n=1}^{3} C_{mn} \alpha_{nr} S_n \left( \psi_{jr} - \phi_r + \tau \right)$$
$$- \sum_{\substack{n=4\\n \neq j}}^{N} C_{mn} \alpha_{nr} S_n \left( \psi_{jr} - \psi_{nr} + \tau \right)$$
(9)

Here, both  $Z_{\text{ECG}}$  and  $Z_{\text{other}}$  represent the difference between the time-shifted ECG segments **V** and the various source components, after they have been time-shifted and scaled. Similar results as in Eq. (7) can be obtained for the estimation of  $\hat{C}$  and  $\hat{\alpha}$  [9]. For the estimation of  $\hat{\psi}$  and  $\hat{\phi}$ , a similar approach as used for **S**, *C*, and  $\alpha$  can no longer be adopted as both  $\psi$  and  $\phi$  are part of the argument of **S**, leading to complex solutions. As an alternative, the quadratic form of *Q* in Eq. (5) can be minimized for varying the values for  $\psi$ and  $\phi$  only, while keeping the values for *C*, **S**, and  $\alpha$  fixed. The minimization of *Q* implicitly maximizes the probability distribution  $p(C, \mathbf{S}, \alpha, \psi, \phi | \mathbf{V})$ .

By expanding the square in Eq. (5) and omitting terms that do not vary with either  $\psi_{ij}$  or  $\phi_j$ , the minimization of Q can be expressed as the maximization of:

$$Y_{\text{ECG}} = \sum_{m=1}^{M} \sum_{t=1}^{T} \left[ \left( \sum_{n=1}^{3} C_{mn} \alpha_{nj} S_n \left( t - \phi_j \right) \right) \times \left( V_{mj} \left( t \right) - \sum_{n=4}^{N} C_{mn} \alpha_{nj} S_n \left( t - \psi_{nj} \right) \right) \right]$$
(10)



Fig. 2. Example of a recorded fetal ECG signal (top graph), the estimated ECG template (center graph), and the filtered ECG signal (bottom graph). The detected peaks in the filtered signal are indicated by triangles. Note that due to immaturity of the fetal heart, an irregular heart beat pattern can be seen here.

and

$$Y_{\text{other}} = \sum_{m=1}^{M} \sum_{t=1}^{T} \left[ C_{mi} \alpha_{ij} S_i (t - \psi_{ij}) \times \left( V_{mj} (t) - \sum_{\substack{n=1\\n \neq i}}^{3} C_{mn} \alpha_{nj} S_n (t - \phi_j) - \sum_{\substack{n=4\\n \neq i}}^{N} C_{mn} \alpha_{nj} S_n (t - \psi_{nj}) \right) \right].$$
(11)

The maximization of Eq. (10) with respect to  $\phi_j$  can be regarded as the maximization of the cross-correlation between the combined ECG sources and the data after subtraction of the other sources. The maximization of Eq. (11) with respect to  $\psi_{ij}$  can be regarded as the cross-correlation between a single source  $S_i$  and the data after subtraction of the other sources.

The algorithm for estimating the template for the periodic waveform that can be used in the matched filter constitutes the consecutive calculation of  $\hat{\mathbf{S}}$  according to Eq. (7), of  $\hat{\alpha}$  and  $\hat{C}$  with similar expressions as for  $\hat{\mathbf{S}}$ , and the calculation of  $\hat{\phi}$  and  $\hat{\psi}$  via the maximization of Eq. (10) and (11).

#### C. Matched filtering and heart rate detection

Since the first three sources are modeled to represent the ECG, the first three rows of **S** and the first three columns of **C** can be used to generate *M* ECG templates  $\hat{V}(t)$  for the recorded signals:

$$\hat{V}_{m}(t) = \sum_{n=1}^{3} C_{mn} S_{n}(t) \,. \tag{12}$$

By convolving the templates  $\hat{V}(t)$  with the recorded signals and applying a peak detection algorithm as described in [4], the heart rate can be inferred. In Fig. 2 the performance of the developed LVSS method on a low-quality fetal ECG recording is exemplified.



Fig. 3. Two examples of the application of the LVSS method and wavelet processing to enhance the SNR of ECG signals. In both (a) and (b) the top signal represents the ECG signal with additive noise, the center signal results from processing by the LVSS method, and the bottom signal results from wavelet processing (4-level Daubechies-10 decomposition). For (a) the SNR of the ECG signal is 8 dB, for (b) this is -8 dB. Note that the original ECG signals used for generating the noisy ECG examples are identical for (a) and (b).

## III. EVALUATION ON LOW-QUALITY ECG SIGNALS

The LVSS method is evaluated by applying it on 12lead (M = 12) ECG signals of ten different patients, each 2 minutes long, sampled at 1 kHz, and each corrupted with additive measurement noise at a variety of signal to noise ratios (SNR's). These signals were obtained from the Physionet database [10], together with annotations by experts. The ECG signals are segmented into half-overlapping, 1 second long epochs. For comparison, the SNR of the same ECG recordings is also enhanced through wavelet-based denoising, using various wavelets (i.e. Coiflet, Daubechies, and Symlet) at various levels (i.e. 1 to 7) and the Matlab wavelet analysis toolbox (The Mathworks, Inc).

In Fig. 3 two signals are shown, one with SNR of 8 dB and one with SNR of -8 dB. For both signals the resulting signals after processing by the LVSS method and processing by a level-4 Daubechies-10 wavelet.

The examples shown in Fig. 3 demonstrate that both the LVSS method and the wavelet method can significantly improve the SNR of the ECG signals. For the ECG signal with relatively large SNR (Fig.3(a)), the wavelet method slightly outperforms the LVSS method in terms of the SNR of the resulting signal. Vice versa, for the ECG signal with relatively small SNR (Fig. 3(b)), the LVSS method outperforms the wavelet method. Specifically, in the center graph of Fig. 3(b) the peaks in the signal are still visible. For the bottom graph, they are however no longer visible and peak detection will be complicated.

Given the fact that the LVSS method performs significantly better than the wavelet method for low SNR recordings and performs sufficiently well for high SNR recordings, the LVSS method seems a proper choice for use in the detection of the heart rate. This statement is confirmed in Fig. 4. In this figure, all wavelet methods mentioned above



Fig. 4. SNR of the output signal as a function of the SNR of the input signal. The results of the LVSS method are depicted with the solid line, the results of the wavelet method are depicted with the dashed line. Since the wavelet method performs similarly for all wavelets, for clarity only the result for the Coiflet-1 wavelet is depicted. Note that these results would very much overlap with the results for other wavelets. The results for both the LVSS and wavelet method are averaged over all patients. For illustration of the performance of both methods, also the SNR of the original, uncorrupted ECG signal is depicted as the dotted line.

and the LVSS method are applied on the ECG signals of all ten patients and the performance is expressed in terms of the SNR of the method's output signals. The results of these analyses are depicted in Fig. 4. Due to the fact that the LVSS method does not provide the original ECG signal (with less noise), but provides a transformed representation of the ECG, it is impossible to determine which part of the transformed signal constitutes "signal" and which part "noise". The standard definition of SNR can, hence, not be used. With the main interest in this paper being the detection of the heart rate, the SNR is here defined as the ratio between the mean peak-to-peak amplitude of the QRS complexes and the RMS amplitude of the complete ECG signal  $V_{\rm rms}$ : SNR =  $\overline{V}_{\rm pp}/V_{\rm rms}$ , where  $\overline{V}_{\rm pp}$  indicates the mean of the various individual  $V_{pp}$  values. The positions of the QRS complexes are based on expert annotations in the original ECG signals (i.e. the ECG signals without additive noise).

From Fig. 4 it can be seen that for recordings with high input SNR the wavelet method even returns ECG signals with a higher SNR. The reason for this is that with almost no noise present, the wavelet output still contains the QRS complexes (i.e.  $V_{pp}$  does not change) but has reduced the amplitude of other parts of the ECG (i.e.V<sub>rms</sub> is reduced). As discussed before, the LVSS method performs acceptable for high input SNR and outperforms the wavelet methods for low input SNR. The main reason for the weaker performance with high input SNR is that physiological waves in the segments between QRS complexes are affected by the matched filter in such way that the  $V_{\rm rms}$  is increased. In contrast to the wavelet methods for which  $V_{\rm rms}$  increases with lower input SNR, however, for the LVSS method  $V_{\rm rms}$  does not further increase with decreasing input SNR as additional noise in the input signal is effectively suppressed by the matched filter.

#### IV. DISCUSSION & CONCLUSIONS

In this paper a method is developed for robust estimation of the heart rate in low-quality ECG signals. The method operates by using latency-variable source separation to generate a template of the periodical signal and subsequently applying this template as a matched filter. The performance of the method is evaluated by comparing it to the performance of a wavelet-based signal enhancement method. This comparison shows that for ECG signals with a relatively large SNR, the wavelet-based method performs better, but that for low-SNR ECG signals, the LVSS method performs better. Since the performance of the LVSS method for high-SNR ECG signals is sufficiently well to allow for detection of the heart rate, the LVSS method seems more suitable for use in heart rate detection algorithms. More data is, however, needed for more conclusive statement about the performance of either method.

Besides using it for processing of ECG signals, the LVSS method has the potential to be applied on any other periodical signal without the need for adaptation of the method. As a side effect, any preprocessing of the periodical signal (e.g. high-pass filtering to suppress respiratory signals) – as long as it does not compromise the periodicity – can be applied to further improve the LVSS method. For example, the ECG signals can first be enhanced using the wavelet-based method before further enhancing them using the LVSS method. Future research would hence include the use of preprocessing methods for LVSS and the evaluation of the method on other periodical signals. In addition, the potential improvement in the robustness of the method by combining the heart rate detected from the matched filtered signal with the estimated latencies also needs to be further studied.

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