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EEG source localization based on Multivariate Autoregressive Models using Kalman Filtering

J.I. Padilla-Buriticá[1], E. Giraldo[2] and G. Castellanos-Domínguez[1].
[1] Universidad Nacional de Colombia, Manizales, Colombia, [2] Universidad Tecnológica de Pereira, Pereira, Colombia

Abstract— **The estimation of current distributions from electroencephalographic recordings poses an inverse problem, which can approximately be solved by including dynamical models as spatio-temporal constraints onto the solution. In this paper, we consider the electrocardiography source localization task, where a specific structure for the dynamical model of current distribution is directly obtained from the data by fitting multivariate autoregressive models to electroencephalographic time series. Whereas previous approaches consider an approximation of the internal connectivity of the sources, the proposed methodology takes into account a realistic structure of the model estimated from the data, such that it becomes possible to obtain improved inverse solutions. The performance of the new method is demonstrated by application to simulated electroencephalographic data over several signal to noise ratios, where the source localization task is evaluated by using the localization error and the data fit error. Finally, it is shown that estimating MVAR models makes possible to obtain inverse solutions of considerably improved quality, as compared to the usual instantaneous inverse solutions, even if the regularized inverse of Tikhonov is used.**

I. INTRODUCTION

Estimation of the brain activity from electroencephalographic (EEG) measurements is known to be an ill-posed inverse problem (infinite number of different current sources arise to identical scalp recordings) that cannot be solved without some kind of regularization. Electroencephalographic Source Localization (ESL) is a technique that consists in inferring the internal configuration of the brain which could explain the electromagnetic activity reflected in the scalp, i.e., the inverse problem in EEG. In this regard, multivariate autoregressive model (MVAR) can be proposed to carry out the ESL task, because of the benefits presented by a parametric model such as the accuracy and the ability to track the time dynamics of a variable [1].

Nonetheless, the description obtained from electrodes with MVAR parameters does not correspond to the same activity within the brain, so these models are not sufficient to address or describe all the internal dynamics of the sources [2]. For this reason, it is mandatory to improve the representation of the internal connectivity of the sources estimated directly from data. In this way, one technique for solving the EEG inverse problem is Kalman filtering, because it provides a natural framework for incorporating dynamic EEG constraints in source localization.

The non-uniqueness of the inverse problem implies that assumptions on the source model, as well as anatomical and physiological. Therefore, a priori knowledge about the source region should be taken into account to obtain a unique solution, i.e. static case [3]. In order to overcome such a drawback, a new method has been recently proposed that takes into account the dynamics of EEG based on physiological linear or nonlinear models as a dynamic constraint to improve the solution of the inverse problem [4], [5]. However, the resulting dynamic inverse problem solution can not describe real interaction among sources. Therefore, in order to improve the dynamic solution, it is necessary a realistic dynamical model that describes the neuronal activity using for example autoregressive models.

This paper presents a methodology based on MVAR model with time invariant parameters, that can be used jointly with the Kalman filter to estimate the source dynamics, wich is implemented on a realistic model of brain computed with the boundary element method (BEM). The performance of the technique is demonstrated by application to simulated EEG signals for different levels of noise where the source localization task is evaluated using the localization error and the data fit error. Finally it is shown that despite employing the regularized pseudo inverse of Tikhonov, MVAR parameters provide sufficient information to achieve a precise location.

II. MATERIALS AND METHODS

A. Dynamic MVAR Modeling

Consider a measured EEG signal $\boldsymbol{y}[k] = [y_1[k], y_2[k] \cdots y_n[k]]^\mathsf{T}$, $\boldsymbol{y} \in \mathbb{R}^{n\times 1}$; where n is the number of EEG channels at time instant k. It is possible to represent the time series $\boldsymbol{y}[k]$ through a multivariate dynamical model defined as:

$$\begin{aligned}\boldsymbol{y}[k] &= \sum_{i=1}^{p} \boldsymbol{A}_i \boldsymbol{y}[k-i] + \boldsymbol{\eta}[k] \\ &= \boldsymbol{A}_1 \boldsymbol{y}[k-1] + \cdots + \boldsymbol{A}_p \boldsymbol{y}[k-p] + \boldsymbol{\eta}[k] \qquad (1)\end{aligned}$$

where $\boldsymbol{A}_i \in \mathbb{R}^{n\times n}, i = 1,\dots,p$, represents the MVAR parameter matrices, vector $\boldsymbol{\eta}[k] \sim \mathcal{N}(0, C_{\boldsymbol{\eta}})$ represents the non-modeled features of the system, i.e. observation noise, with covariance matrix $C_{\boldsymbol{\eta}} \in \mathbb{R}^{n\times n}$. In this study, the parameter matrices associated with the dynamic behavior (see (1)) are to be estimated directly from the EEG signals.

The relation between the EEG and the neural activity into the brain can be defined as follows:

$$\boldsymbol{y}[k] = \boldsymbol{M}\boldsymbol{x}[k] + \boldsymbol{\eta}[k] \qquad (2)$$

where $\boldsymbol{x}[k] \in \mathbb{R}^{3m\times 1}$ is the current density associated with the neuronal activity being m the number of distributed sources inside the brain. Besides, the lead field matrix $\boldsymbol{M} \in$

$\mathbb{R}^{n\times 3m}$ relates the current density inside the brain with the EEG measurements $\boldsymbol{y}[k]$ and can be computed using Maxwell equations for a specific head model.

Additionally, it is possible to assume that the dynamic behavior associated with $\boldsymbol{x}[k]$ is similar to the one presented in $\boldsymbol{y}[k]$. Therefore, applying the relation (2) in Eq. (1), the MVAR model for $\boldsymbol{x}[k]$ is reformulated as:

$$\boldsymbol{M}\boldsymbol{x}[k] = \sum_{i=1}^{p} \boldsymbol{A}_{\mathrm{i}}\boldsymbol{M}\boldsymbol{x}[k-i] \tag{3}$$

Regularized inverse of $\boldsymbol{M}$ defined as $\boldsymbol{M}^{-1} \in \mathbb{R}^{3m\times n}$ can be found by Tikhonov method as follows

$$\boldsymbol{M}^{-1} = \left(\boldsymbol{M}^{\mathsf{T}}\boldsymbol{M} + \lambda^2\boldsymbol{I}\right)^{-1}\boldsymbol{M}^{\mathsf{T}} \tag{4}$$

where the regularization parameter λ is chosen using methods of parameter selection, by example the L-curve approach [6]. However, when $\boldsymbol{M}$ is ill-conditioned, this approach can lead to a significant decrease in numerical stability of the inverse. Instead, QR factorization is used in the Tikhonov regularization method of Eq. (4) to obtain solutions having better numerical stability [7].

Using the regularized inverse, Eq. (3) is reformulated as:

$$\boldsymbol{x}[k] = \sum_{i=1}^{p} \boldsymbol{M}^{-1}\boldsymbol{A}_{\mathrm{i}}\boldsymbol{M}\boldsymbol{x}[k-i] + \boldsymbol{\varepsilon}[k]$$

and defining the parameters matrices $\boldsymbol{F}_i = \boldsymbol{M}^{-1}\boldsymbol{A}_i\boldsymbol{M}$, and being $\boldsymbol{\varepsilon}[k] \in \mathbb{R}^{3m\times 1}$ an additive random variable defined as $\boldsymbol{\varepsilon}[k] \sim \mathcal{N}(0, \boldsymbol{M}^{-1}\boldsymbol{C}_\eta\boldsymbol{M})$. As a result, the linear dynamic model associated with the neural activity is defined as:

$$\boldsymbol{x}[k] = \sum_{i=1}^{p} \boldsymbol{F}_i\boldsymbol{x}[k-i] + \boldsymbol{\varepsilon}[k] \tag{5}$$

Consequently, a dynamic forward problem for EEG simulation can be formulated using two equations: a discrete time measurement equation as defined in (2) and the dynamic state space equation proposed in (5). The advantage of this approach is that the dynamical model is estimated directly from the measured EEG signals $\boldsymbol{y}[k]$, and is related through $\boldsymbol{M}$ with a model for neural activity $\boldsymbol{x}[k]$. Equations (2) and (5) can be reformulated as a first order model, as follows:

$$\begin{cases} \boldsymbol{z}[k] &= \boldsymbol{F}\boldsymbol{z}[k-1] + \boldsymbol{B}\boldsymbol{\varepsilon}[k] \\ \boldsymbol{y}[k] &= \boldsymbol{M_e}\boldsymbol{z}[k-1] + \boldsymbol{\eta}[k] \end{cases} \tag{6}$$

where $\boldsymbol{z}[k-1] = [\boldsymbol{x}[k-1]^{\mathsf{T}}, \boldsymbol{x}[k-2]^{\mathsf{T}}, \cdots, \boldsymbol{x}[k+p-1]^{\mathsf{T}}]^{\mathsf{T}}$, $\boldsymbol{B} = [I, 0, \cdots, 0]^{\mathsf{T}}, \boldsymbol{M_e} = [\boldsymbol{M}, \boldsymbol{0}, \cdots, \boldsymbol{0}]$ and

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F_1} & \boldsymbol{F_2} & \cdots & \boldsymbol{F_p} \\ \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \ddots & \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}$$

Equation (6) is further used in the formulation of the dynamic inverse problem, as proposed in [5].

III. Dynamic inverse problem

Equation (6) becomes the ill-posed dynamic version of the inverse problem for the linear case, which is equivalent to the maximum a posterior (MAP) estimate when the statistics are Gaussian [8], as follows:

$$\hat{\boldsymbol{z}}[k] = \arg\max_{\boldsymbol{z}[k]}\{\rho_{\boldsymbol{z}[k]|\boldsymbol{y}[1],\ldots,\boldsymbol{y}[k]}\} \tag{7}$$

where $\rho_{\boldsymbol{z}[k]|\boldsymbol{y}[1],\ldots,\boldsymbol{y}[k]}$ is a conditional density. MAP estimation looks for the current estimate $\hat{\boldsymbol{z}}[k]$ that is most probable, given both the model $\boldsymbol{F}$ as well as the set of measurements $\boldsymbol{y}[1], \ldots, \boldsymbol{y}[k]$.

The solution to Eq. (7) is to obtain through the following recursion, where the equations to generate the prior mean and covariance from the posterior are often referred as the time update equations of the Kalman filtering, and the equation to generate posteriors from the priors are referred as the measurement update equations. Just before measuring $\boldsymbol{y}[k]$ our state of knowledge is described by the prior mean and covariance of $\boldsymbol{z}[k]$. After measuring $\boldsymbol{y}[k]$, we compute in sequence the time update equations:

$$\hat{\boldsymbol{z}}[k]^- = \boldsymbol{F}\hat{\boldsymbol{z}}[k-1] \tag{8a}$$
$$\boldsymbol{C}[k]^- = \boldsymbol{F}\boldsymbol{C}[k-1]\boldsymbol{F}^{\mathsf{T}} + \boldsymbol{C_\eta} \tag{8b}$$

where $\hat{\boldsymbol{z}}[k]^-$ is defined as a priori estimation of $\hat{\boldsymbol{z}}[k]$, and $\boldsymbol{C}[k]^-$ is defined as a priori covariance. Then, we compute

$$\boldsymbol{G}[k] = \boldsymbol{C}[k]^-\boldsymbol{M}^{\mathsf{T}}\left(\boldsymbol{M}\boldsymbol{C}[k]^-\boldsymbol{M}^{\mathsf{T}} + \boldsymbol{M}^{-1}\boldsymbol{C}_\eta\boldsymbol{M}\right)^{-1} \tag{9a}$$
$$\hat{\boldsymbol{z}}[k] = \hat{\boldsymbol{z}}[k]^- + \boldsymbol{G}[k]\left(\boldsymbol{y}[k] - \boldsymbol{M}\hat{\boldsymbol{z}}[k]^-\right) \tag{9b}$$
$$\boldsymbol{C}[k] = \left(\boldsymbol{I} - \boldsymbol{G}[k]\boldsymbol{M}\right)\boldsymbol{C}[k]^- \tag{9c}$$

For a linear model with Gaussian noise statistics the Kalman filter produces the optimal estimates $\hat{\boldsymbol{z}}[k]$. Because the mean and covariance completely specify a Gaussian density function, the Kalman filter effectively estimates the conditional density $\rho_{\boldsymbol{z}[k]|\boldsymbol{y}[1],\ldots,\boldsymbol{y}[k]}$ at each time.

IV. Results and discussion

A. Experimental Setup

When using an MVAR model, at least, two main issues are to be solved: the parameter estimation and the optimal selection of model order. To perform the former task, the MVAR parameters are estimated by using the Kalman filter, due to the precision of the reconstruction of the signals obtained with this method [9]. Regarding the latter issue, it has been taken the model order that minimizes the Bayesian Information Criterion BIC, given by,

$$BIC(p) = -2\ln|\boldsymbol{C_e^2}| + 2p\ln(N). \tag{10}$$

where $\boldsymbol{C_e^2}$ is the estimation error covariance matrix of parameters $\boldsymbol{A_i}, i = 1, \ldots, p$, and N is the number of signal samples.

To develop a comparative analysis between the proposed method and the static method, a simulation experiment is carried out. The system dynamics is approximated through second order linear time invariant model taking into account

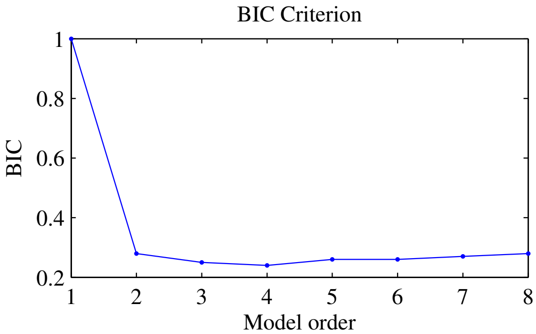

Fig. 1. Model order selection using BIC

anatomic constraints related with spatial coupling between sources. Thus, the dynamic inverse problem solutions are compared for a given set of simulated EEG trials. In this way, the following errors are used to evaluate the performance: localization error and data fit, defined as:

– Localization error:

$$L_e = ||\boldsymbol{r}_{dip} - \hat{\boldsymbol{r}}_{dip}||_2 \tag{11}$$

where $||.||_2$ is the L_2 norm, $\boldsymbol{r}_{dip}$ and $\hat{\boldsymbol{r}}_{dip}$ are original dipole and estimate dipole positions respectively.

– Data fit error:

$$D_f = \frac{1}{T}\sum_{k=1}^{T}\frac{||\boldsymbol{y}[k] - \hat{\boldsymbol{y}}[k]||}{||\boldsymbol{y}[k]||} \tag{12}$$

where $\boldsymbol{y}[k]$ and $\hat{\boldsymbol{y}}[k]$ are original EEG and estimated EEG respectively.

B. Simulated EEG recordings

A major issue regarding EEG source localization is obtaining its meaningful evaluation, Particularly, because the true source locations are not available if testing with real EEG data; therefore a reliable estimation of reconstruction error poses a challenge. Thus, the most common approach is to use simulated EEG data where underlying neural activity sources are known. For this purpose, one source randomly located into the brain is considered for simulation. The generation of simulated EEG dataset requires for selecting a model of the brain dynamics, which should display a proper complex spatio–temporal behavior. Here, the temporal dynamics are suggested to be simulated using a second order linear model comprising one sine function, which are applied in one source with frequency in the alpha band (namely, 10 Hz). Specifically, the simulated brain dynamics are generated by using the following time model structure:

$$\boldsymbol{x}_k = \boldsymbol{A}_1\boldsymbol{x}[k-1] + \boldsymbol{A}_2\boldsymbol{x}[k-2] + \boldsymbol{\varepsilon}[k] \tag{13}$$

where the process noise $\boldsymbol{\varepsilon}[k]$ holds the artificial harmonic function with sampling rate of value 1 kHz, $\boldsymbol{A}_1 = a_1\boldsymbol{I} + b_1\boldsymbol{L}$ and $\boldsymbol{A}_2 = a_2\boldsymbol{I}$, being $\boldsymbol{I} \in \mathbb{R}^{3N\times 3N}$ the identity matrix. Notation $\boldsymbol{L} \in \mathbb{R}^{3N\times 3N}$ stands for the matrix operator that represents the spatial interaction among sources [4], [5]. The next values are assumed as the initial set of parameters: $a_1 = 1.2$, $b_1 = 0.05$, $a_2 = -0.9$, which had been empirically fixed in [5]. Besides, in accordance with the measurement model given in Eq. (13), 20 synthetic EEG data trials of one–second length are generated from the simulated current densities in Eq. (13) by multiplication with the lead field matrix $\boldsymbol{M}$. For achieving the methodology robustness, the additive noise term $\boldsymbol{\varepsilon}$ is given for several signal noise ratios (SNR) (namely, 5, 10, 15, 20, 25, and 30 dB) [10]. Prior to computing an inverse solution, a discretized solution space is defined as a regular grid of dimension $10\times10\times10$ mm uniformly located sources into the brain. At each source, the 3D local current density vector is mapped, as usual, to the 33 electrode sites for the $10-20$ standard international system. The solutions are computed over a realistic head model calculated with the boundary element method described in [11].

C. Estimation Results

TABLE I

MEAN DATA-FIT ERROR (%) FOR DYNAMIC MODEL WITH TIME INVARIANT PARAMETERS

Approach	Source	5 dB	15 dB	25 dB
MVAR-KF	surface	2.75 ± 0.8	1.76 ± 0.93	1.32 ± 0.78
	deep	2.84 ± 0.74	2.26 ± 0.87	1.59 ± 0.83
STATIC	surface	3.25 ± 0.96	1.84 ± 0.77	1.84 ± 0.92
	deep	4.96 ± 0.73	3.86 ± 0.8	2.76 ± 0.56

In Table I, it can be seen consistent results with those presented in [3] which used a spheric head model for the static-case. When a MVAR model with Kalman filter is used, the performance of the estimator is improved in case of superficial and deep sources compared to the static case. Furthermore, the performance of the algorithm decreases as the source to be found is deep, because it captures the dynamics of nearby sources. The Fig. 2 shows that the proposed methodology captures the dynamics of all sources, presented with larger original source simulated and dispersion around it. The Fig. 4(c) shows that the dynamic method has less dispersion that obtained by the static method of the Fig. 4(b). Fig. 3 displays values of the grand mean of localization error, accomplished for both cases of dynamic and static modeling, where the following notations stand for compared methods of estimation: static inverse problem (static), and dynamic MVAR (dyn). It should be quoted that attained error values for the baseline static case are consistent with the ones assessed when using a spheric head model in [3]. As seen, the MVAR model outperforms the static performance, specially, in case of low SNRs.

To make clear the influence of the discussed estimation methods of dynamic neural activity based on MVAR models, Fig. 4(a), 4(c) and 4(b) depicts the ESL at a time instant for the original activity, the estimated activity using the MVAR model and the estimated activity using the static inverse problem, respectively. The mapping is carried out for a deep source using a three layer realistic head model, which is computed based on the Boundary Element Method. It can be seen that the dynamic estimation method of Fig. 4(c), lead to an improved reconstruction of the neural activity, although

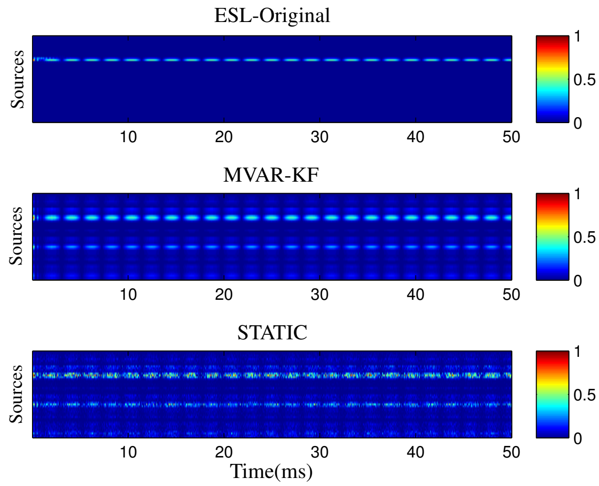

Fig. 2. Source comparison

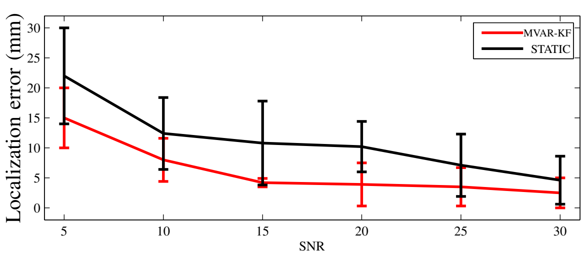

Fig. 3. Localization error for several SNRs

the static method considerably increases the amplitude at the neighborhood sources compared with the true activity.

V. Conclusions

This paper addresses the problem in EEG source localization, using a second order multivariate autoregressive model with time invariant parameters coupled with the Kalman filter. The obtained results show a better performance in terms of the localization error and data fit error against the static case for several noise ratio. In comparison with the dynamic methods proposed in [5], an improvement in the selection of the model is achieved since it is estimated directly from the data. Even when a regularized inverse is used to find the model matrices, the estimated model represents the relations among sources more adequately than generalized models. As future work, a simplified dynamic nonlinear model structure should be estimated directly from the data obtaining a non uniform model of the brain for a better representation of internal connectivity among sources.

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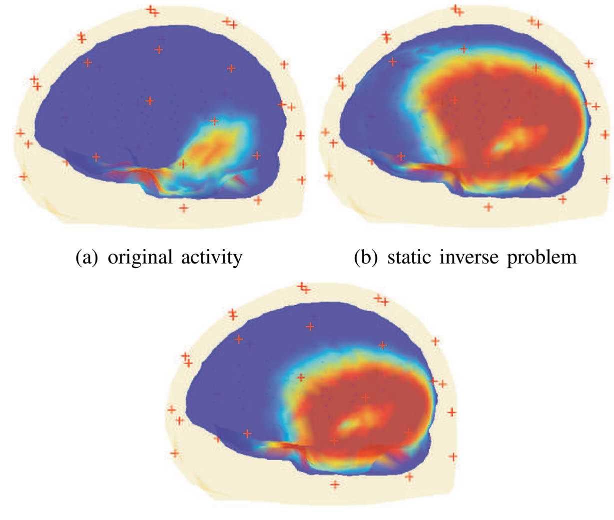

(a) original activity (b) static inverse problem

(c) MVAR

Fig. 4. 3D mapping for original and estimated neural activity

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