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# Stability of a Double Inverted Pendulum Model during Human Quiet Stance with Continuous Delay Feedback Control

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*Abstract*—**Recent debate about neural mechanisms for stabilizing human upright quiet stance focuses on whether the active and time delay neural feedback control generating muscle torque is continuous or intermittent. A single inverted pendulum controlled by the active torque actuating the ankle joint has often been used for the debate on the presumption of well-known ankle strategy hypothesis claiming that the upright quiet stance can be stabilized mostly by the ankle torque. However, detailed measurements are showing that the hip joint angle exhibits amount of fluctuations comparable with the ankle joint angle during natural postural sway. Here we analyze a double inverted pendulum model during human quiet stance to demonstrate that the conventional proportional and derivative delay feedback control, i.e., the continuous delay PD control with gains in the physiologically plausible range is far from adequate as the neural mechanism for stabilizing human upright quiet stance.**

## I. Introduction

Upright posture during human quiet stance is mechanically unstable due to a high position of the total center of mass ($CoM_{total}$) of the human body, represented as a system of multi-link rigid bodies. It has been suggested that quiet stance is controlled and stabilized mostly by the ankle joint torque [1], a mechanism which is known as ankle strategy of postural control. Since the passive stiffness of the ankle joint, arising from mechanical viscoelasticity of the ankle joint, is smaller than the rate of growth of the gravitational toppling torque [2], [3], active torque generated by neural feedback control is required for stabilizing the upright posture, a necessary neural mechanism, unfortunately affected by a significant neural feedback transmission delay. Those classical observations have been supporting, for a long time, mathematical modeling of human quiet stance with a single-link inverted pendulum that is actuated by the passive as well as the active torques at the ankle joint.

The recent debate about neural stabilization of human upright quiet stance focuses on whether the active, delayed neural feedback control generating muscle torque is continuous or intermittent [4]-[6]. The inverted single pendulum model controlled by the active torque at the ankle joint has often been used for the debate. However, detailed measurements are showing that the hip joint angle exhibits amount of fluctuations comparable with the ankle joint angle during natural postural sway [7], [8], suggesting that the dynamics of the standing posture could not be accurately captured by the single inverted pendulum model. For example, the angular displacement at the hip is significantly greater than the angular displacement at the ankle, confirming that hip joint motion cannot be ignored [7]. Moreover, hip joint rotations might support the postural system in minimizing the acceleration of the $CoM_{total}$ [8]. Since the ankle joint torque as well as the stiffness of the ankle joint required for stabilizing the quiet stance can be different in the two biomechanical models (single-link vs. multi-link inverted pendulum model) [9], the analysis of the multi-link inverted pendulum model in the human quiet stance is crucial for characterizing in a correct way both control strategies, i.e., continuous or intermittent control hypotheses.

Here we construct a double inverted pendulum model during quiet stance in the sagittal plane. Two joints of the model correspond to the ankle and hip. For each joint, we assume that the active torque is generated by a continuous and time delayed proportional and derivative (PD) feedback controller as in the conventional stiffness control model of the upright stance [10], [11]. In the single inverted pendulum model, the upright posture can easily be destabilized if the derivative gain of the PD control is small when we assume that the proportional gain of the PD control is large enough to supplement the insufficient passive stiffness of the ankle, known as delay-induced instability [4], [6]. Thus, models with the stiffness control hypothesis [11] usually assume a large derivative gain to avoid delay-induced instability, as well as a large proportional gain, which might be physiologically implausible [12]. In this study, we analyze the double inverted pendulum model during human quiet stance to demonstrate that the conventional PD delay feedback control, with the gains in the physiologically plausible range, is far from adequate as the neural mechanism for stabilizing human upright quiet stance.

## II. Models and Methods

The double inverted pendulum model during quiet standing in the sagittal plane is shown in Fig. 1a. The upper and lower links of the model correspond to the following segments: head-arm-trunk ($Link_{HAT}$) and leg ($Link_L$), respectively. The distal end of $Link_L$ is fixed in the space by a pin joint ($Joint_a$),

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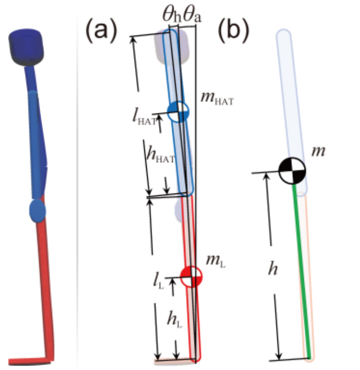


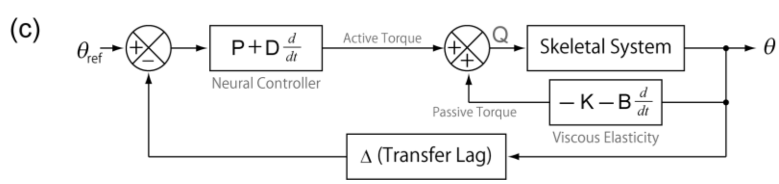


Fig. 1. A double inverted pendulum model during quiet standing. (a) The upper and lower links of the model correspond to the head-arm-trunk and the lower extremity, respectively. Distal end of the lower link corresponds to the ankle joint. Joint between the lower and upper links corresponds to the hip joint. (b) $m$: total mass of double pendulum. $h$: distance from the ankle joint to the total center of mass of the double pendulum with fully extended hip joint. (c) Block diagram of neuro-musculo-skeletal system model with a neural feedback transmission delay Δ in the feedback path. $\theta_{ref}$ =(0,0) in this study.

which corresponds to the ankle joint. The proximal end of $Link_L$ and the distal end of $Link_{HAT}$ are also connected by a pin joint ($Joint_h$), which corresponds to the hip joint. The joint angles and parameters of the link masses, lengths, and the distances from the joint to CoM of each link are defined as in Fig. 1a. The parameter values in Table I are for an adult with 1.7 m of height and 60 kg of weight, using the segment ratio as used in the previous study [13]. In Fig. 1b, $m$ represents the total mass of the double pendulum, and $h$ the distance from $Joint_a$ to $CoM_{total}$ of the double pendulum when $Joint_h$ is fully extended. The block diagram in Fig. 1c represents the neuro-musculo-skeletal system for the inverted double pendulum model that assumes the conventional continuous feedback controller. Motion equation of the double inverted pendulum model is described as;

$$\mathbf{M}\ddot{\boldsymbol{\theta}} + \mathbf{G}\boldsymbol{\theta} = \mathbf{Q} \tag{1}$$

where $\boldsymbol{\theta}$ is the joint angle vector, $\mathbf{M}$ the inertia matrix, $\mathbf{G}\boldsymbol{\theta}$ the gravitational torque vector, and $\mathbf{Q}$ the joint torque vector. Because the tilt angles and angular velocities during quiet stance are small, we approximate as $\sin\theta_a \approx \theta_a$, $\cos\theta_a \approx 1$, $\sin\theta_h \approx \theta_h$, $\cos\theta_h \approx 1$, and $O(\omega^2) \approx 0$ with $\omega$ being the angular velocity. Terms of the centrifugal force and Coriolis force vanish by this linearization. The joint torque vector depends on how we model the passive and the active torques.

### A. Stability Analysis of the Model without Active Torque

We first consider the double inverted pendulum model whose joints are actuated only by the passive torque. The passive joint torque vector is modeled as follows.

$$\mathbf{Q} = \begin{pmatrix} \tau_a \\ \tau_h \end{pmatrix} = \begin{pmatrix} \tau_a^{passive}(\theta_a, \omega_a) \\ \tau_h^{passive}(\theta_h, \omega_h) \end{pmatrix} = \begin{pmatrix} -K_a\theta_a - B_a\omega_a \\ -K_h\theta_h - B_h\omega_h \end{pmatrix}$$

where $K_a$, $B_a$, $K_h$, and $B_h$ are the passive elastic coefficient and viscosity coefficient of the ankle joint and those of the hip joint, respectively. Motion equation of the model is described by the following linear ordinary differential equation.

$$\frac{d}{dt}\begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \mathbf{M}^{-1}\left(-\mathbf{G} + \begin{pmatrix} -K_a & 0 \\ 0 & -K_h \end{pmatrix}\right) & \mathbf{M}^{-1}\begin{pmatrix} -B_a & 0 \\ 0 & -B_h \end{pmatrix} \end{pmatrix} \begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} = \mathbf{A}_{passive}\begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} \tag{2}$$

The eigenvalues of $\mathbf{A}_{passive}$ determine the stability of the model, which can be obtained as the solutions $\lambda$ of

$$\left|\lambda\mathbf{I} - \mathbf{A}_{passive}\right| = 0 \tag{3}$$

where $\mathbf{I}$ is the 4×4 identity matrix. Stability of the upright posture depends on the four parameters ($K_a$, $B_a$, $K_h$, $B_h$). For a given set of values of these parameters, the system is stable if the real parts of all eigenvalues are negative. Otherwise the system is unstable, meaning that the upright posture of the model without active feedback torque cannot be stabilized. Here we fix both $B_a$ and $B_h$ at 4 Nms/rad. For a variety of values of $K_a$ and $K_h$, we calculate solutions of (3) to show how the stability of the model depends on the values of $K_a$ and $K_h$.

### B. Stability Analysis of the Model with Active Torque

We consider the double inverted pendulum model whose joints are actuated by the passive torque as well as the active torque generated by the continuous time delay PD feedback controllers. In this case, the joint torque vector is defined as follows.

$$\mathbf{Q} = \begin{pmatrix} \tau_a \\ \tau_h \end{pmatrix} = \begin{pmatrix} \tau_a^{passive}(\theta_a, \omega_a) + \tau_a^{active}(\theta_a, \omega_a) \\ \tau_h^{passive}(\theta_h, \omega_h) + \tau_h^{active}(\theta_h, \omega_h) \end{pmatrix} = \begin{pmatrix} -K_a\theta_a - B_a\omega_a - P_a\theta_{a\Delta} - D_a\omega_{a\Delta} \\ -K_h\theta_h - B_h\omega_h - P_h\theta_{h\Delta} - D_h\omega_{h\Delta} \end{pmatrix}$$

where $P_a$, $D_a$, $P_h$, and $D_h$ are the proportional and derivative gains of the active feedback control for the ankle joint and those for the hip joint, respectively. The subscript Δ is the time delay due to the neural transmission time, i.e., $\theta_\Delta = \theta(t-\Delta)$, $\omega_\Delta = \omega(t-\Delta)$. (See Fig. 1c). In this study, Δ is set to 0.1 s. Then, motion equation of the model in this case becomes the following delay differential equation:

$$\frac{d}{dt}\begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} = \mathbf{A}_{passive}\begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\begin{pmatrix} -P_a & 0 & -D_a & 0 \\ 0 & -P_h & 0 & -D_h \end{pmatrix} \end{pmatrix} \begin{pmatrix} \theta_{a\Delta} \\ \theta_{h\Delta} \\ \omega_{a\Delta} \\ \omega_{h\Delta} \end{pmatrix} = \mathbf{A}_{passive}\begin{pmatrix} \theta_a \\ \theta_h \\ \omega_a \\ \omega_h \end{pmatrix} + \mathbf{A}_{active}\begin{pmatrix} \theta_{a\Delta} \\ \theta_{h\Delta} \\ \omega_{a\Delta} \\ \omega_{h\Delta} \end{pmatrix} \tag{4}$$

By substituting a solution of (4) with the form of ($\theta_a(t)$, $\theta_h(t)$, $\omega_a(t)$, $\omega_h(t)$)$=e^{\lambda t}$($\theta_a([0,-\Delta])$, $\theta_a([0,-\Delta])$, $\omega_a([0,-\Delta])$, $\omega_h([0,-\Delta])$) into (4), we obtain the following equality.

TABLE I
PARAMETERS OF DOUBLE INVERTED PENDULUM MODEL

| Symbol | Description | Values |
|---|---|---|
| $m_L$ | Segment mass of $Link_L$ | 60 × 0.35 kg |
| $l_L$ | Length of $Link_L$ | 1.70 × 0.51 m |
| $h_L$ | Distance from distal end to center of mass of $Link_L$ | 1.70 × 0.255 m |
| $m_{HAT}$ | Segment mass of $Link_{HAT}$ | 60 × 0.62 kg |
| $l_{HAT}$ | Length of $Link_{HAT}$ | 1.70 × 0.45 m |
| $h_{HAT}$ | Distance from distal end to center of mass of $Link_{HAT}$ | 1.70 × 0.225 m |
| $\theta_a$ | Rotation angle of $Joint_a$ | --- rad |
| $\theta_h$ | Rotation angle of $Joint_h$ | --- rad |
| $\omega_a$ | Angular velocity of $Joint_a$ | --- rad/sec |
| $\omega_h$ | Angular velocity of $Joint_h$ | --- rad/sec |
| Δ | Delay in the feedback loop | 0.1 sec |

$$\lambda e^{\lambda t}\begin{pmatrix}\theta_a([0, -\Delta])\\ \theta_h([0, -\Delta])\\ \omega_a([0, -\Delta])\\ \omega_h([0, -\Delta])\end{pmatrix} = \left(e^{\lambda t}\mathbf{A}_{\text{passive}} + e^{\lambda t}e^{-\lambda\Delta}\mathbf{A}_{\text{active}}\right)\begin{pmatrix}\theta_a([0, -\Delta])\\ \theta_h([0, -\Delta])\\ \omega_a([0, -\Delta])\\ \omega_h([0, -\Delta])\end{pmatrix}$$

The solution $\lambda$ which satisfies this relationship, except a trivial solution with zero initial value, can be calculated from the following transcendental equation.

$$\left|\lambda\mathbf{I} - \mathbf{A}_{\text{passive}} - e^{-\lambda\Delta}\mathbf{A}_{\text{active}}\right| = 0 \qquad (5)$$

Equation (5) includes quadratic terms of $\lambda$ and exponential functions of $\lambda$, implying that there is an infinite number of solutions. We look at four dominant eigenvalues to determine the stability of the model. Dynamics of the model with the active feedback torque is characterized by the eight parameters ($K_a$, $B_a$, $K_h$, $B_h$, $P_a$, $D_a$, $P_h$, $D_h$). Here we set $K_a$ as $0.8mgh$, $B_a$ as 4 Nms/rad, and $P_a$ as $0.4mgh$. A set of ($K_h$, $B_h$) is set to either ($0.6mgh$, 4) or ($1.0mgh$, 10). The following values of $D_a$ and $D_h$ are examined; 10 Nms/rad, 30 Nms/rad, and 50 Nms/rad for $D_a$, and 5 Nms/rad, 10 Nms/rad, and 20 Nms/rad for $D_h$. There are eighteen combinations of the parameter values for ($K_h$, $B_h$, $D_a$, $D_h$). We calculate root locus diagrams of the model as a function of the parameter $P_h \in [0.2mgh, 1.0mgh]$ for each set of ($K_h$, $B_h$, $D_a$, $D_h$), by which we obtain four root loci for the four dominant eigenvalues. If all of four roots for a value of $P_h$ have negative real parts, upright posture of the model under active torque with that set of the parameter values can be stabilized. If any roots is in the right-half plane, the model cannot be stabilized by the active control.

## III. Results

### A. Stability of the Model without Active Torque

Stability of the model without the active feedback torque in the $K_a$-$K_h$ plane, where $K_a \in [0, 2.0mgh]$ and $K_h \in [0, 1.0mgh]$ with $B_a$ and $B_h$ being fixed at 4 Nms/rad, is shown in Fig. 2. Pair of values of the $K_a$, $K_h$ parameters that fall in the red region correspond to conditions of instability. The blue region is an area of stability. Note that a critical elastic coefficient altering the stability for the equivalent single inverted pendulum, whose mass is $m$ and distance from the ankle joint to $\text{CoM}_{total}$ is $h$, is $1.0mgh$. Fig. 2 shows that $K_a$ should be larger than $1.0mgh$ for the stability of the upright posture of the double inverted pendulum. Moreover, when $K_h$ is small, even if $K_a$ is large, the upright posture of the double inverted pendulum cannot be stabilized.

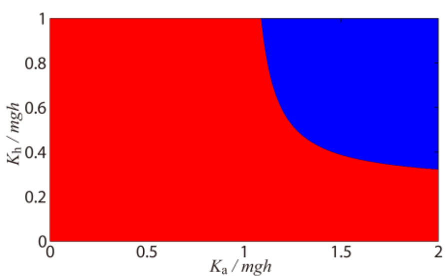


Fig. 2. Stability region of the double inverted pendulum model without the active feedback torque in the ($K_a$, $K_h$) plane. Values of both $B_a$ and $B_h$ are fixed at 4 Nms/rad. If a set of value of ($K_a$, $K_h$) is located in the red region, the upright posture of the model without the active torque is unstable. If it is in the blue region, the upright posture of the model without the active torque is stable.

### B. Stability of the Model with Active Torque

The root locus diagrams (RL) as a functions of $P_h$ for the model with the active feedback torque are shown in Fig. 3, in which $K_a$, $B_a$ and $P_a$ are fixed at $0.8mgh$, 4 Nms/rad, and $0.4mgh$, respectively. RL has four branches: two branches, referred to as $\text{RL}_{in}$, are close to the origin and packed in a small region; the other two branches, referred to as $\text{RL}_{anti}$, are relatively far from the origin and span a longer range. Blue curves in Fig. 3 represent $\text{RL}_{anti}$ for the model with ($K_h$, $B_h$)=($0.6mgh$, 4); in the case of red curves ($K_h$, $B_h$)=($1.0mgh$, 10). White and black circles on each branch correspond, respectively, to $P_h=0.2mgh$ and $P_h=1.0mgh$.

When ($K_h$, $B_h$)=($0.6mgh$, 4), for all three values of $D_h$, the blue $\text{RL}_{anti}$ stays in the right half-plane for any values of $P_h \in [0.2mgh, 1.0mgh]$, meaning that the equilibrium point of the model is unstable regardless of the values of $P_h$. For ($K_h$, $B_h$)=($1.0mgh$, 10), when $D_h$=10 Nms/rad and 20 Nms/rad, the model is unstable for any value of $P_h$ as shown by the red $\text{RL}_{anti}$ in the center and right columns of Fig. 3. Figure 4 shows magnifications of three panels in Fig. 3 left-column around their origin when $D_h$=5 Nms/rad. In this case, when $D_a$=10 Nms/rad, $\text{RL}_{in}$ stays in the right half-plane, meaning that the model is unstable. When $D_a$=30 and 50 Nms/rad, $\text{RL}_{in}$ stays in the left half-plane. When $D_a$=30 Nms/rad, two roots on the $\text{RL}_{anti}$ branch cross the imaginary axis at $P_h=0.24821mgh$. When $D_a$=50 Nms/rad, two roots on the $\text{RL}_{anti}$ cross the imaginary axis at $P_h=0.20385mgh$. Thus, the upright posture of the model with ($K_h$, $B_h$)=($1.0mgh$, 10) is stable if $P_h$ is between $0.2mgh$ and $0.24821mgh$ for $D_a$=30 Nms/rad, and if $P_h$ is between $0.2mgh$ and $0.20385mgh$ for $D_a$=50 Nms/rad.

## IV. Discussion

In this study, we construct a double inverted pendulum model during human quiet stance, and analyze the stability of the model with and without active, time delay, and continuous neural feedback control. The results of the study show that the model without active torque can be stable only if the viscoelasticity of each of two joints is very large, which might be far from the physiological range. The model with the active torque can be stabilized if the eight parameter values of the ankle and hip joint torques, i.e., $K_a$, $B_a$, $K_h$, $B_h$, $P_a$, $D_a$, $P_h$, and $D_h$, are set appropriately. However, the ranges of the parameter values that can stabilize the upright posture of the model with the active torque are quite limited, meaning that the continuous and time-delay PD feedback control cannot stabilize the double inverted pendulum in a robust manner.

Throughout this study, we assume $K_a=0.8mgh$, $B_a$=4 Nms/rad, and $P_a=0.4mgh$. For the model with the active feedback torque, the upright posture with the following sets of the parameter values is stable: Case (A) $K_h=1.0mgh$, $B_h$=10 Nms/rad, $D_a$=30 Nms/rad, $P_h \in [0.2mgh, 0.24821mgh]$, $D_h$=5 Nms/rad. Case (B) $K_h=1.0mgh$, $B_h$=10 Nms/rad, $D_a$=50 Nms/rad, $P_h \in [0.2mgh, 0.20385mgh]$, $D_h$=5 Nms/rad.

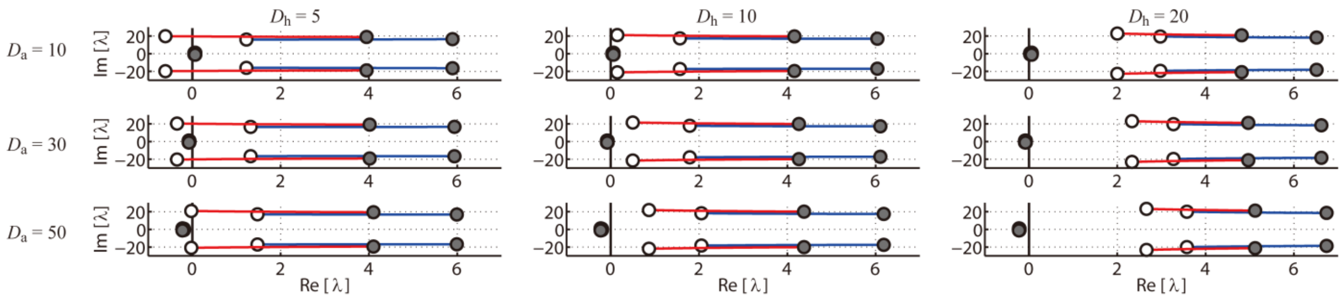


Fig. 3. Root locus diagrams as a function of the proportional gain of hip joint $P_h$ ($\in$ [0.2 *mgh*, 1.0 *mgh*]) of the active feedback. For each, the passive ankle stiffness, viscosity, and proportional gain of the active feedback torque at ankle joint are constant; $K_a$=0.8*mgh*, $B_a$=4, $P_a$=0.4*mgh*. Blue curves are the root locus for ($K_h$, $B_h$)=(0.6*mgh*, 4). Red curves are the root locus for ($K_h$, $B_h$)=(1.0*mgh*, 10). ○ indicates $P_h$=0.2*mgh*, and ● indicates $P_h$=1.0*mgh*.

Comparing Case (A) with Case (B), we see that the stability range of $P_h$ decreases as the value of $D_a$ increases unlike the case of the single inverted pendulum. Indeed, the model with $D_a$ larger than 50 Nms/rad cannot be stabilized by the active feedback torque. This means that the upper value of $D_a$ that can stabilize the upright posture is also limited.

Let us discuss how the double pendulum with the active feedback control behaves in detail. It can be confirmed that $\theta_a$ and $\theta_h$ coordinates of the eigenvector for the pair of complex roots on the $RL_{in}$ branch have always the same sign. Moreover, $\theta_a$ and $\theta_h$ coordinates of the eigenvector for the pair of complex roots on $RL_{anti}$ branch have opposite signs. Thus, the transient oscillation mode of the double pendulum associated with the roots on the $RL_{in}$ branch represents the motion in which $Link_L$ and $Link_{HAT}$ moves together, i.e., with an in-phase relationship. The other oscillation mode associated with the roots on the $RL_{anti}$ branch represents the motion in which $Link_L$ and $Link_{HAT}$ moves in opposite direction, i.e., with an anti-phase oscillation. In experimental observations of postural sway, the in-phase and the anti-phase motions may be considered roughly as the sway around the ankle joint and the hip joint, respectively. In both cases, the imaginary part of the pair of complex roots represents the oscillation frequency. The oscillation frequency of anti-phase mode in the model is about 6 Hz, which is much faster than 1.45 Hz for the hip joint fluctuations reported in the previous experimental study [14]. Comparison between the imaginary part of the pair of the complex roots for the model with ($K_h$, $B_h$)=(0.6*mgh*, 4) and that with ($K_h$, $B_h$)=(1.0*mgh*, 10) shows that the oscillation frequency decreases as the passive viscoelasticity of the hip joint decreases. This means that the passive viscoelasticity of the hip joint should be much lower than the values that we used in this study, if we assume the continuous active PD feedback control as the neural mechanism. If, however, the passive viscoelasticity of the hip joint is much lower than the values used in this study, the upright posture of the model cannot be stabilized for any proportional and derivative gains of the active and continuous feedback controller, suggesting that the continuous PD feedback might not be the neural mechanism that the human central nervous system employs for the postural control.

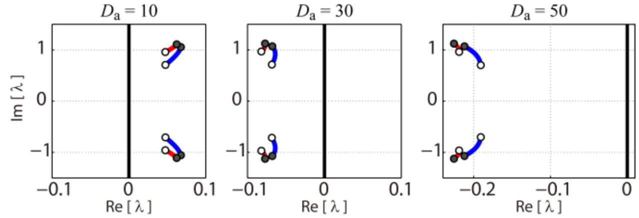


Fig. 4. The three panels are magnifications of parts of root loci in Fig. 3 near the origin for $D_h$=5 (three panels in Fig. 3-left column).

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