

# Eye Position Prediction in the Case of Nystagmus and Refixations

Craig O. Savage

**Abstract**—Nystagmus is a condition in which the eyes move in an uncontrolled fashion. These eye movements confound attempts to identify patient fixation, as desired eye positions cannot be maintained. We aim to estimate the eye positions in the case of refixations and superimposed uncontrolled motion. By incorporating saccade detection and resetting our estimation algorithms, we are able to track the nystagmus motion independently of fixation direction. We employ our algorithm on data collected from patients with latent/manifest-latent and pendular nystagmus conditions.

## I. INTRODUCTION

Nystagmus is an oscillatory condition of the eyes, consisting either of slow, grossly sinusoidal oscillations or of alternating slow movements and faster ones which return the eyes to the point of intended fixation. It may be classified as to the time of onset, motion direction, and waveform type. The incidence of nystagmus, in all of its forms, has been estimated to be about 24 in ten thousand [1]. People who are either born with nystagmus or develop the condition early in life frequently suffer from reduced visual acuity [2]. Infrequently, people with congenital forms of nystagmus also suffer from oscillopsia [3]. Oscillopsia is the apparent motion of the world, even when there is no such movement [4], [5]. Normally sighted people may experience similar feelings after spinning for a time then stopping - in addition to dizziness, the apparent spinning of the world is like oscillopsia.

In cases when nystagmus is acquired later in life, oscillopsia is common [6]; in fact, oscillopsia has been called the “characteristic symptom” of acquired nystagmus [7]. Oscillopsia is extremely disabling, both in its reduction of visual acuity and its interference of how the affected patient can interact with the apparently moving environment. Medical treatments of acquired nystagmus, thereby treating resulting oscillopsia, include botulum toxin (a.k.a. botox) [7], gabapentin and memantine [8], and other drugs dependent upon the type and cause of nystagmus [9], [10], [11].

Furthermore, upon losing their sight, many people develop nystagmus [12], [13]. However, as these people lose their vision, the impact of the nystagmus is secondary to the loss of eyesight.

C. O. Savage is a research fellow in the Faculty of Electrical and Electronic Engineering, University of Melbourne, Australia, 3010 cosavage@unimelb.edu.au

External treatments of nystagmus have also been attempted. Possible means include a special combinations of glasses and contacts [14], compensation for eye movements by controlled optics [15], or opening and closing shutters [16]. Such adaptive measures report success with reducing oscillopsia for patients with pendular nystagmus, but not other forms such as jerk nystagmus [16]. Possible reasons include the less regular nature of nonpendular nystagmus.

In an attempt to predict more complex forms of nystagmus, we employ the Kalman filter [17]. The Kalman filter has been employed for studying eye movements, including smooth pursuit [18], saccade detection [19], and prediction of eye movements in normally sighted individuals [20]. Here, we attempt to utilize the filter to differentiate between fixation direction and uncontrolled eye movements. The Kalman filter is employed on real data collected from eye movements of two patients, one of which had a pendular nystagmus, the other with a latent/manifest-latent nystagmus. The waveform of the patient with latent/manifest latent nystagmus is similar to waveforms arising from jerk nystagmus.

The remainder of the paper is organized as follows. Section II describes our implementation of the Kalman filter for this problem. The methodology of collecting data from patients is described in Section III. A presentation of results is given in Section IV. Finally, we discuss implications of the results and consider potential future work in Section V

## II. TRACKER MODELS

To conserve space, we consider tracking the horizontal component of the motions of a single eye. Extending the methods here to jointly track horizontal and vertical components of both eyes may be treated in a similar manner, so long as appropriate care is taken in regards to correlations between eyes, and directions within a single eye. For notational purposes, we matrix (or vector) values as a bold letter, with estimates having a “hat.” We consider discrete time steps, with the current time being indexed as  $k$ , and the upcoming time as  $k+1$ . Subscripts denote the time of estimate validity, conditioned upon all information up to and including the second subscript. For example,  $\hat{\mathbf{x}}_{k+1|k}$  is the estimate of  $\mathbf{x}$  at time  $k+1$ , conditioned upon all information to time  $k$ , inclusive.

### A. Kalman filter

We estimate the eye positions based upon a Kalman filter [17]. The state space includes position ( $x$ ) and velocity ( $\dot{x}$ ) information,

$$\mathbf{x} = [x, \dot{x}]^T. \quad (1)$$

The state is assumed to evolve as

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{w}_k \quad (2)$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad (3)$$

where  $\Delta t$  is the time between samples, and  $\mathbf{w}_k$  is a vector of independent, identically distributed random variables of mean zero, and covariance  $E[\mathbf{w}_k \mathbf{w}_k^T] = \mathbf{Q}$ . We model eye movements as having a random, constant acceleration during time steps, resulting in

$$\mathbf{Q} = \sigma_a^2 \begin{bmatrix} \Delta t^4/4 & \Delta t^3/2 \\ \Delta t^3/2 & \Delta t^2 \end{bmatrix} \quad (4)$$

where  $\sigma_a^2$  reflects the size of the accelerations. Measurements of the state are given by

$$z_{k+1} = \mathbf{H}\mathbf{x}_{k+1} + v_{k+1} \quad (5)$$

with  $\mathbf{H} = [1, 0]$ , and  $v$  is a random noise component reflecting uncertainty in the measurement, with mean zero and variance  $E[v^2] = R$ . Note for this case,  $z, v$  are scalar. All noise terms are assumed to be uncorrelated, independent and identically distributed normal random variables with known covariance.

The state estimate is initialized with a two-point procedure. That is, assume that  $z_1, z_2$  have been received. The recursion begins with

$$\hat{\mathbf{x}}_{2|2} = \begin{bmatrix} z_2, \frac{z_2 - z_1}{t_2 - t_1} \end{bmatrix}^T. \quad (6)$$

The initial estimated state error covariance (ESEC), denoted as  $\mathbf{P}$ , is initialized as the  $2 \times 2$  identity matrix ( $\mathbf{I}_2$ ), which covers uncertainty in both position and velocity.

The ESEC and state estimates are then propagated via the standard Kalman filter method.

### B. Filter reset

We identify saccades based upon the magnitude of the difference between the predicted and received measurements, also referred to as the innovation, which was used in [19]. We employ a simpler procedure - if two consecutive innovations are greater than a threshold, we reset the filter according to a similar procedure as initialization. That is, assume that at

times  $t_n, t_{n-1}$  the innovation was greater than a threshold. Then,

$$\hat{\mathbf{x}}_{n|n} = \begin{bmatrix} z_n, \frac{z_n - z_{n-1}}{t_n - t_{n-1}} \end{bmatrix}^T \quad (7)$$

$$\mathbf{P}_{n|n} = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

in a manner analogous to filter initialization, only at general times  $n - 1, n$  instead of 1, 2.

## III. DATA COLLECTION

Data were collected from two patients at the Department of Optometry at the University of Melbourne. Study participants were admitted following all applicable steps for ethical data collection. One patient showed signs of acquired pendular nystagmus, the other had a latent/manifest-latent nystagmus. Data were collected at 1000 Hz using a Microguide 1000, which records horizontal eye movements in the form of an analog signal [21], and advertises a 1 arc minute sensitivity. Data were resampled to 50 Hz to simulate data rates available from head mounted systems. Examples of patient waveforms are shown in Figure 1. In total, two minutes of data were collected for each patient, and included multiple shifts in gaze direction during the test.

## IV. RESULTS

For eye position estimation, the following values were employed:

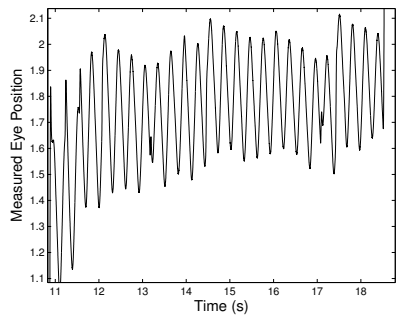
$$\begin{aligned} \sigma_a &= 100 \\ R &= 1/60 \end{aligned}$$

where the value of  $\sigma_a = 100$  is consistent with [20], and  $R$  arises from the advertised sensitivity of the system used for measuring eye movements. The threshold for refixation was set to 0.5; that is, two consecutive measurements had innovations greater than 0.5 in magnitude, the filter was reset, according to the methodology in Section II-B. Example results are shown in Figure 2.

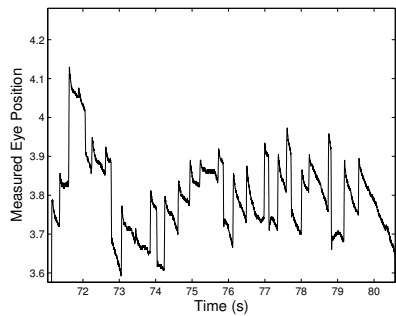
Eye position predictions are made from the Kalman filter recursion. At each time  $k$ , the predicted eye position is given by

$$\hat{\mathbf{x}}_{k+1|k} = \mathbf{F}\hat{\mathbf{x}}_{k|k} \quad (9)$$

The position component of this predicted state is plotted along with the measurements in Figure 2. Two sets of predictions are plotted, with and without filter resets as described in Section II-B.



(a) Recorded eye movements from patient one in this study. The nystagmus is largely pendular.



(b) Recorded eye movements from patient two in this study. The patient was diagnosed with latent/manifest latent nystagmus. The eye waveform is similar to that of jerk nystagmus.

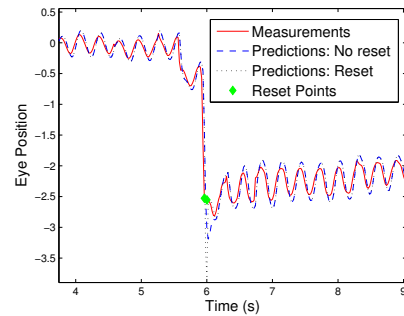
Fig. 1. Eye movement waveforms for the patients in this study.

## V. CONCLUSIONS AND FUTURE WORKS

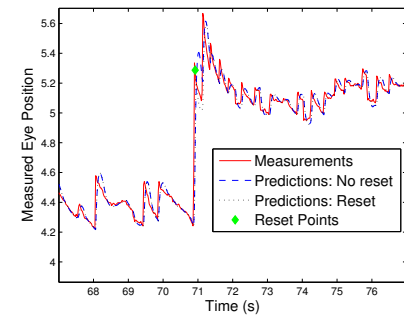
### A. Conclusion

We have demonstrated tracking efficacy on recorded eye movements on patients with nystagmus, including areas where patients refixate. The tracker has done surprisingly well considering that neither nystagmus waveform had been modeled explicitly. Two different formulations have been developed and perform similarly in predicting eye movements, and have different advantages and disadvantages. Restarting the estimation recursion based upon refixation classification shows a small difference in estimated position shortly after a saccade. However, due to the relative weights between measurement and process precision, the two estimates are nearly identical after less than second. Values for measurement and process noise were taken from the literature (based on system specifications and [20]); tuning the filter according to different noise values will produce different estimations.

Furthermore, filter resetting may occur at multiple points during gaze shifts. During some refixations, measurements fell during the fast portion, resulting in multiple resets.



(a) Measurements and eye position predictions with and without filter reset, for patient 1.



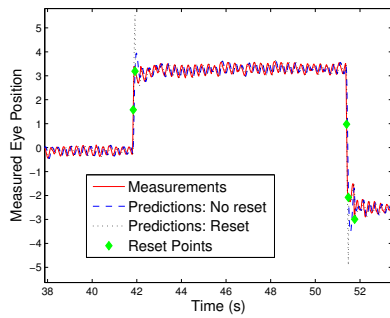
(b) Measurements and eye position predictions with and without filter reset, for patient 2.

Fig. 2. Tracking results for both patients, using both methods. The trackers perform similarly for both state spaces, and both nystagmus types.

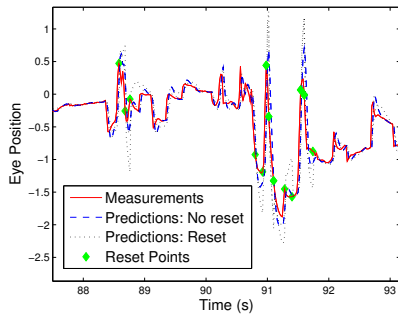
Although the saccades seem instantaneous in the figures, the total time is about 0.1s, or  $\sim 5$  samples at a measurement rate of 50 Hz. Uncertainty between whether the patient was having difficulty refixating or whether nystagmic motion had an increased amplitude was apparent during tracker analysis. Examples of these difficulties are presented in Figure 3.

### B. Future works

Future work could include data collection under different conditions, such as utilization of a portable eye tracker. Many of the deviations between the track and the measurements are below advertised measurement accuracy; it is hard to say which is a “better” estimate of true eye movement. Tracker parameters could be “tuned” to more accurately reflect individual patient conditions. Saccade identification measures could be strengthened by threshold determination, the two-step procedure of [19], using sustained eye velocity as a detector, or incorporating more information on saccade probabilities based upon typical behavior. Differentiating saccades due to refixations as opposed to as part of a jerk nystagmus waveform is difficult, as the two motions share muscular phenomenology.



(a) The filter performed multiple reinitializations during a single saccade, as the saccade duration was longer than a sample time.



(b) The filter resets during nystagmic saccades and abrupt direction changes for patient 2.

Fig. 3. Example difficulties encountered with filter reset.

On the clinical side, work at preventing oscillopsia, similar to [15], [16], could be attempted by running a Kalman filter in real-time. Clinical studies could be used to infer whether the track information is preferred over measurement information by testing both as a means to preventing oscillopsia.

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