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Distributed Parameter Statics of Magnetic Catheters

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***Abstract*—We discuss how to use special Cosserat rod theory for deriving distributed-parameter static equilibrium equations of magnetic catheters. These medical devices are used for minimally-invasive diagnostic and therapeutic procedures and can be operated remotely or controlled by automated algorithms. The magnetic material can be lumped in rigid segments or distributed in flexible segments. The position vector of the cross-section centroid and quaternion representation of an orthonormal triad are selected as DOF. The strain energy for transversely isotropic, hyperelastic rods is augmented with the mechanical potential energy of the magnetic field and a penalty term to enforce the quaternion unity constraint. Numerical solution is found by 1D finite elements. Material properties of polymer tubes in extension, bending and twist are determined by mechanical and magnetic experiments. Software experiments with commercial FEM software indicate that the computational effort with the proposed method is at least one order of magnitude less than standard 3D FEM.**

I. Introduction

A magnetic catheter is an interventional device containing permanent or permeable magnets, navigated in a patient's body lumens by steering at the distal tip using an external magnetic field $\boldsymbol{B}$ and by changing the inserted length L at the proximal end by an advancer. In contrast to manually manipulated catheters which are bent at the tip using pull wires and twisted by a handle at the proximal end to change the plane of bending, magnetic catheters do not need torque transmission via their shaft since the field can rotate the tip. Combined with a localization system that measures tip position and orientation, an X-ray or ultrasound imaging system, other physiological sensors such as ECG, pressure or force, end-effectors such as ablation electrodes, biopsy probes and a software platform that adds visualization, automation and control, these catheters can be teleoperated by human physicians at a workstation placed in a control room which may be adjacent to the surgical room or placed in a remote location. Mechanical modeling of these devices is valuable for many practical purposes: Visual feedback of virtual device to the operator, simulation for training and pre-procedure planning, design optimization and automatic control. With the help of varying degrees of automation implemented in firmware and software, the efficacy and efficiency of minimally-invasive medical procedures may potentially be improved with likely less risk for the patient and less X-ray exposure for the physician.

Typically, the body of a catheter is made from an extruded thermoplastic polymer which may be irradiated to increase cross-linking, making it more “elastomeric.” The body may be reinforced with coils or braids to improve kink resistance and pushability and loaded with a radiopaque substance for fluoroscopy. While this work uses a cardiac electrophysiology catheter with a polymer body as the subject, similar elongate devices such as guide wires, endoscopes, sheaths and other application areas such as the vasculature, lungs, kidneys, brain etc. can be considered by selecting appropriate materials (e.g. polymer, steel, nitinol) and by tailoring the dimensions and construction of structural and magnetic parts. The modeling method described herein applies to all such elongate devices using rigid or flexible electromagnets or permanent magnets whose volumetric magnetization is independent of the external field, hence excludes permeable materials. These magnetic materials may be placed at arbitrary intervals or distributed with axially varying density along the device body. However, constitutive relationships between stress and strain of the selected materials must be determined experimentally after they have been formed into a part. For instance, for polymers, these relationships are not isotropic due to molecular chain layout and the extrusion process; they depend heavily on temperature and sometimes on water absorption if uncoated; and manufacturer's specifications for a sample material in terms of Young's modulus and shear modulus or Poisson's ratio are not adequate for precise engineering purposes. Polymers are viscoelastic and they can go into the plastic range if care is not taken. Furthermore, when a polymer is reinforced with a metal coil or braid, properties of the assembled part cannot be easily calculated from those of its constituents.

Here we will restrict our model to spatially uniform, externally generated magnetic fields. Time variation of the actuation ($\boldsymbol{B}$ field plus advancer) and surrounding tissues are assumed to be much slower than the dynamics of the catheter. Therefore, static equilibrium configurations can approximate a moving time average of dynamic configurations and seem to be adequate for most medical catheterization applications. Our formulation follows the special Cosserat rod theory described in [1], [2] and references therein. The kinematic formulation is intrinsically one-dimensional (1D) in contrast to reduction from 3D elasticity. For our application there are two motivating factors: First, the resulting finite-element model (FEM) uses a 1D domain parametrized by the arc length s in the reference configuration and thus yields significant computational savings compared to 3D FEM; second, it is time consuming and expensive to identify anisotropic material properties of polymers and polymer-metal parts in 3D. Instead, the 1D formulation allows us to

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determine the constitutive relationships between strains and internal forces by experimenting on manufactured (but not assembled) tubing segments.

II. Static Model

When a magnetic catheter with discrete rigid magnets is in free-space, extending from a sheath or support, its centroid curve is planar, there is no twist, and curvatures of homogeneous segments completely determine the equilibrium configuration [3]. When there is contact with tissue only at the tip, then a closed-form solution is still possible using Jacobi elliptic functions [4]. However, for distributed magnetic materials with arbitrary magnetization direction, and when the catheter makes contact at multiple locations, twisting and shear deformation occur in addition to bending. At every point in the domain six degrees-of-freedom (DOF) are required as functions of the reference arc length s. These can be selected as the strain variables: Two for bending $\kappa_1(s), \kappa_2(s)$, two for shear $\nu_1(s), \nu_2(s)$, one for extension $\nu_3(s)$, and one for twist $\kappa_3(s)$. Alternatively, the position vector of the cross-section centroids $\boldsymbol{r}(s)$ and an orthonormal triad $\{\boldsymbol{d}_i(s)\}$, $i = 1, 2, 3$ may be used. First two vectors of the triad are called "directors" and they span the cross-section. Denote by $\cdot'$ differentiation with respect to s. This triad rotates as

$$\boldsymbol{d}_i' = \boldsymbol{\kappa} \times \boldsymbol{d}_i\,, \quad \boldsymbol{\kappa} \stackrel{\text{def}}{=} \sum_i \kappa_i \boldsymbol{d}_i\,. \tag{1}$$

The rotational DOF may also be represented by a proper orthogonal matrix $\boldsymbol{R}(s) \in SO(3)$, transforming the standard basis $\{\boldsymbol{e}_i\}$ of $\mathbb{R}^3$ to the local basis of the directors. In the case of straight rods with negligible shear and extension, the latter method leads to an efficient, custom FE discretization based on beam elements [5], which is particularly useful for control design [6]. However, representing the three rotational DOF with six variables in $\boldsymbol{d}_1, \boldsymbol{d}_2$ and three orthonormality constraints (or nine variables and six constraints for $\boldsymbol{R}$) necessitates special integration methods at every step of the solution to maintain these constraints since standard methods like Runge–Kutta are inappropriate [7]. If one wishes to use commercial or standard FE packages instead, then a useful reduction to four variables is afforded by using quaternions $\mathsf{q} = q_0 + \boldsymbol{q}$, with norm $|\mathsf{q}| \stackrel{\text{def}}{=} \sqrt{q_0^2 + \boldsymbol{q}\cdot\boldsymbol{q}} = 1$. Other alternatives include keeping the directors as main variables, but transforming them to and from quaternions only for the configuration update [8]; expressing strain variables in quaternions and using coordinate projection at every step [9] to enforce the unity constraint; deriving the equations of motion in quaternion algebra and using index reduction for the resulting differential-algebraic system followed by a stiff integration method [10]; using $\boldsymbol{r}$, $\boldsymbol{R}$ and strain variables all together in a two-point shooting scheme for the resulting first order ODE's [11].

We prefer a commercial general-purpose FE environment, such as Comsol® for our rod model not only to save time in programming but also for the ability to couple this structural mechanics problem to other physics (fluid, thermal, etc.). For this reason, we choose quaternions to represent rotations as

$$\begin{aligned}\boldsymbol{R} &= [\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{d}_3] \\ &= \left(q_0^2 - \boldsymbol{q}\cdot\boldsymbol{q}\right)\boldsymbol{I}_{3\times 3} + 2\boldsymbol{q}\otimes\boldsymbol{q} + 2q_0\boldsymbol{Q}\,,\end{aligned} \tag{2}$$

where $\boldsymbol{Q}$ is the skew-symmetric matrix corresponding to $\boldsymbol{q}$. Denoting the alternating symbol by ε, the strain variables in terms of the DOF $(\boldsymbol{r}, \mathsf{q})$ are

$$\kappa_i = \frac{1}{2}\sum_{j,k} \varepsilon_{ijk}\boldsymbol{d}_j' \cdot \boldsymbol{d}_k\,, \tag{3}$$

$$\nu_i = \boldsymbol{r}' \cdot \boldsymbol{d}_i\,. \tag{4}$$

The unity constraint is enforced by adding a penalty term

$$P = \frac{1}{2}\left(q_0^2 + \boldsymbol{q}\cdot\boldsymbol{q} - 1\right)^2 \tag{5}$$

to the total potential energy. Indeed, this approach is not only mathematically convenient but also equivalent to scaling the directors in the deformed configuration corresponding to uniform distention of the cross-section. A hollow tube filled with pressurized fluid, such as blood vessels would have this behavior.

Since catheter tubes are made by extrusion it is reasonable to assume that the material is transversely isotropic. In this case, the most general hyperelastic strain energy per unit reference length, up to quadratic order is [12]

$$\begin{aligned}W = \frac{1}{2}\big[&E^b\left(\kappa_1^2 + \kappa_2^2\right) + E^t\kappa_3^2 \\ &+ E^s\left(\nu_1^2 + \nu_2^2\right) + E^e(\nu_3 - 1)^2\big]\,,\end{aligned} \tag{6}$$

where E^b, E^t, E^s, E^e are stiffness in bending, twist, shear and extension respectively, which may depend on s. If the reference configuration is not straight but has nonzero initial curvatures κ_i^r, then (6) still applies via the substitution $\kappa_i \leftarrow (\kappa_i - \kappa_i^r)$. The constitutive relations for the internal contact force $\boldsymbol{n}$ and moment $\boldsymbol{m}$ vectors are

$$\boldsymbol{n} = \sum_i n_i\boldsymbol{d}_i\,, \quad n_i = \frac{\partial W}{\partial \nu_i}\,, \tag{7}$$

$$\boldsymbol{m} = \sum_i m_i\boldsymbol{d}_i\,, \quad m_i = \frac{\partial W}{\partial \kappa_i}\,. \tag{8}$$

For a rod magnetized in the direction perpendicular to its cross-section with magnetic moment per length $\mu(s)$, the mechanical potential energy per length is

$$U_m = -\mu(s)\boldsymbol{d}_3(s)\cdot\boldsymbol{B}\,, \tag{9}$$

and the (energy) functional to be minimized by $(\boldsymbol{r}, \mathsf{q})$ is

$$S(\boldsymbol{r}, \mathsf{q}) = \int_0^L \left(W + U_m + c_u P\right)\mathrm{d}s\,, \tag{10}$$

in which the constant c_u is selected large. Common boundary conditions are geometric at the proximal end $\boldsymbol{r}(0) = 0$,

$q_0(0) = 1$, $\boldsymbol{q}(0) = 0$ and an applied force or torque at the distal end $s = L$. If there are other external conservative forces or moments acting along the rod body or at the ends, the potential energies corresponding to those can be added to (10). The ODE's describing the static equilibrium are found by standard calculus of variations since $(\boldsymbol{r}, \mathfrak{q})$ are unconstrained except for the boundary conditions at $s = 0$. Indeed, the weak form can be used directly. Symbolic computation in Mathematica® is used for this purpose. These equations were translated to Matlab®, calling Comsol® FE functions to mesh the domain, to discretize via cubic Hermite elements to maintain differentiability, and to solve by repetitively using pure Newton's method in a continuity scheme. Comparison to standard 3D FEM indicated that 40-fold reduction in the total FE DOF is achieved without sacrificing accuracy.

The stiffness parameters in (6) were estimated by experiments for tube segments with uniform material and geometric properties. All tests were done at body temperature after allowing sufficient time for soaking in saline solution. Extension stiffness E^e was estimated in pulling tests with constant elongation rate. Viscoelastic behavior is evident in Fig. 1. The standard 3-parameter spring–dashpot model relating stress σ, strain ϵ and their time rates $\dot{\sigma}, \dot{\epsilon}$

$$\dot{\epsilon} = \frac{\dot{\sigma} + \frac{E_2}{\eta}\sigma - \frac{E_1 E_2}{\eta}\epsilon}{E_1 + E_2} \tag{11}$$

using viscous damping parameter η captures both creep and stress relaxation behavior, although three parameters may not be sufficient for quantitatively describing a particular material. Since we are interested in statics, we fitted the experimental data to the steady-state value of the ramp response of force to elongation. The bending stiffness E^b was estimated by attaching small magnets to one end of a tube and deflecting it in a magnetostatic field. At the strain range we were testing, the linearity of the response was very good and E^b did not depend much on tube length, as seen in Fig. 2. The twist stiffness E^t was estimated by rotating one end of a tube to a fixed angle and measuring the torque at the other end by a precision torque meter. Figure 3 indicates that for short tubes the deflection is going outside the linear range. But for magnetic catheters torsional deformation is fairly small, and the linear fit is sufficient for our purposes. Estimating the shear stiffness E^s is not straightforward because creating a configuration with pure shear deformation with constant strain requires a constant body torque to be applied along the whole length of the tube, not just at the boundaries, which is not practical. Instead, a three-point bend test may be used which will combine shear with planar bending and extension. Using numerical optimization, E^s may be estimated that fits the FE simulation to the experimental data in a least-squares sense.

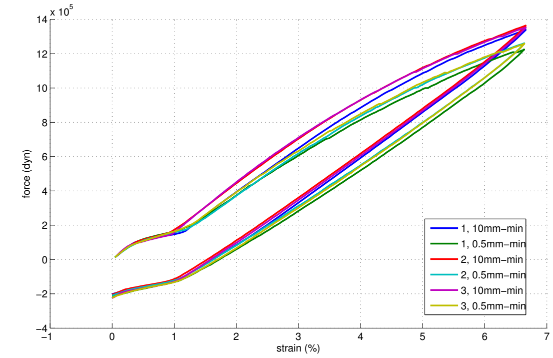

Fig. 1. Extension force (dyn) vs. strain (%) at various elongation rates for catheter tube.

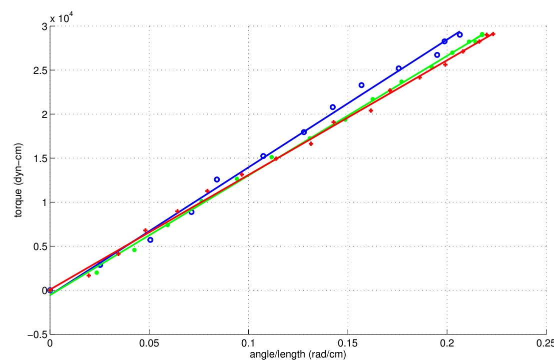

Fig. 2. Magnetic bending torque (dyn–cm) vs. tip angle/length (rad/cm) for tubes of length 3 cm (blue), 4.5 cm (green) and 6 cm (red); experimental results (markers) and fitted lines (solid).

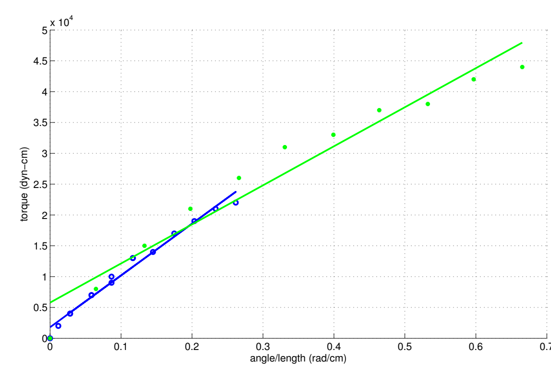

Fig. 3. Twisting torque (dyn–cm) vs. tip angle/length (rad/cm) for tubes of length 6 cm (blue), 2.6 cm (green); experimental results (markers) and fitted lines (solid).

III. Conclusion

For static models of magnetic catheters, the special Cosserat rod-based FE model defined on the 1D domain using arc length as the coordinate yields significant computational savings compared to standard 3D FEM. Using unit quaternions to represent rotational DOF combined with a penalty term added to the mechanical potential energy allows using commercial FE software without resorting to special integration methods that must be employed with rotation matrices. For polymer tubes used in catheters, siffness parameters in bending, extension and twist have been determined from experiments in the strain range required for this application. To the best of our knowledge, there is no standard experiment to determine the shear stiffness of a tubular object and therefore this parameter may be estimated by least-squares fitting of three-point bending data.

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