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3-D Image-guided Diffuse Optical Tomography using Boundary Element Method and MPI Implementation

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Abstract—**Boundary elements provide an attractive method for image-guided multi-modality near infrared spectroscopy in three dimensions using only surface discretization. This method operates under the assumption that the underlying tissue contains piece-wise constant domains whose boundaries are known *a priori* from an alternative imaging modality such as MRI or *micro*CT. This significantly simplifies the meshing process providing both speed-up and accuracy in the forward solution. Challenges with this method are in solving dense matrices, and working with complex heterogeneous domains. Solutions to these problems are presented here, with applications in breast cancer imaging and small—animal molecular imaging.**

I. Introduction

BOUNDARY element method is well-known in heat transfer, fracture mechanics and other engineering problems, for modeling in 3-D using a surface mesh[1-3]. The use of fundamental solutions to the equations allows them to be simplified into a boundary integral representation. Sikora et al[4] and Srinivasan et al[5] applied this method to diffuse optical tomography (DOT) under the assumption that the underlying tissue contains homogeneous or piece-wise constant domains. Zacharopoulos et al[6] carried this further to reconstruct for shapes using spherical harmonics as well as optical properties. However due to the complexity of this problem, this is likely to be limited in accuracy of the recovered optical properties. Srinivasan et al[7] and Ghadyani et al[8] used this method to reconstruct for total hemoglobin, oxygen saturation, water and scatter in-vivo by assuming that the boundaries of the homogeneous domains can be known from MRI, also called image-guided (IG) DOT. In a subject undergoing neoadjuvant chemotherapy, this method showed reduction in total hemoglobin with treatment[7].

The attractiveness of the BEM arises from the fact that volumetric meshing of arbitrary shapes such as arising in breast imaging, is complicated, time consuming, and sometimes, unreliable. Use of surface meshing is simpler, reliable and also easy to automate since several commercial software readily allow surface rendering of tissue shapes after segmentation.

The key challenges facing successful adoption of BEM in image-guided DOT and fluorescence are (1) ability to model heterogeneities, (2) ability to solve large dense matrices arising in BEM forward problem and (3) ability to model distributed sources such as arising in image-guided fluorescence. We have demonstrated a coupled finite element-boundary element for (1) previously[9]. Parallelization offers a potential solution to (2) using open-MP and MPI standards and results show speed-up of up to an order of magnitude in time. For (3), we are currently working on an implementation of BEM combined with dual reciprocity method (BEM-DRM) using compactly supported radial basis functions for approximating fluorescent source. Here we present some of these results.

II. Methods

A. BEM Theory

Under the assumption that the tissue contains homogeneous domains whose boundaries can be obtained by image-segmentation of MRI or CT images, the light diffusion equation simplifies into a modified Helmholtz equation[5]. This can be solved using BEM, when the boundary conditions and the treatment of the source are known. Under

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these conditions, the forward model using boundary integral representation can be written in matrix form as:

$$[A]\{\Phi_i\} - [B]\left\{D_l \frac{\partial \Phi}{\partial n}\right\} = \{Q_i\} \qquad (1)$$

where:

$$A_{i,j} = c_i \delta_{ij} + \oint D_l \frac{\partial G_i}{\partial n} \psi_j ds$$
$$B_{i,j} = \oint G_i \psi_j ds \qquad (2)$$
$$Q_i = \langle q_0, G_i \rangle$$

Here D is the diffusion coefficient, Φ_i is the field, G_i is the Green's function, and q_o is the point source. Here the photon fluence and flux are discretized using linear basis functions ψ_i defined on the triangles of the surfaces.

For DOT, a point source was assumed which does not need volume discretization. Type III boundary conditions were used for the outer boundary taking into account refractive index mismatch between tissue and air[10]. For inner boundaries, continuity conditions on fluence and flux were enforced.

B. Coupled FE-BEM Theory

In the coupled model, finite elements (FE) is used to model spatially varying tissues such as tumors inside the domain. This is incorporated by separating boundary and interior nodes of such tissues, and enforcing continuity conditions on the boundary nodes. Details can be found elsewhere[9]. This can be easily extended to multi-region problems, where the tissue domain is divided into homogeneous regions and heterogeneous regions, and appropriate boundary conditions applied.

C. Parallelization

Equation 1 can be written as:

$$[K]\{x\} = \{b\} \qquad (3)$$

where K is the stiffness matrix containing *[A]* and *[B]* from (1), and x contains the fluence and flux at the boundary nodes. K is a dense matrix that has to be inverted to solve for x; K becomes sparse as the number of subzones in the tissue domain increases. In inverting equation (3), the time for computation scales as N^3, where N is the number of nodes in the surface mesh. This process is also memory-intensive. Approaches such as fast multipole methods have been studied to counter this[11, 12], but require computing multipole expansions and may affect the accuracy of the solution.

Here, we have studied the implementation of message passing interface (MPI) and open-MP standard, to deal with this. The MPI implementation uses parallel LAPACK libraries for solving equation (3) using LU Decomposition. Open-MP uses multiple processors, but is limited to a single machine, and cannot pass the data between machines. This limits the memory available for the solver. MPI, on the other hand, communicates between machines, and hence can be used to extend the size of the problem being solved.

III. Results

A. Coupled FE-BEM

The coupled FE-BEM was applied to six test cases generated from a breast mesh after image-segmentation of MRI from a subject diagnosed with infiltrating ductal carcinoma. Fig. 1 shows the time of computation from three forward models (BEM, FEM and coupled), on the six test cases. On five of the test cases, the coupled method was faster than FEM, with the ability to model heterogeneity. It was found that when the ratio of surface nodes to volume nodes was less than 20%, the coupled method was faster[9].

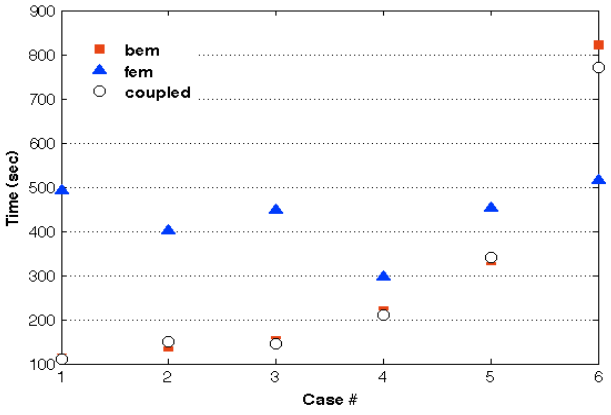

Fig. 1: Time of computation of light fluence from BEM, FEM and coupled models, from six test cases.

B. Parallelization

The results for time of computation obtained from open-MP implementation for three different mesh sizes is plotted in Fig. 2 for different # of cores used. Since open-MP is restricted to one machine, the maximum # of cores possible in this case is 8. The time of computation using 8 cores was less than 21% of the time taken by one core, giving a speed-up of ~80%.

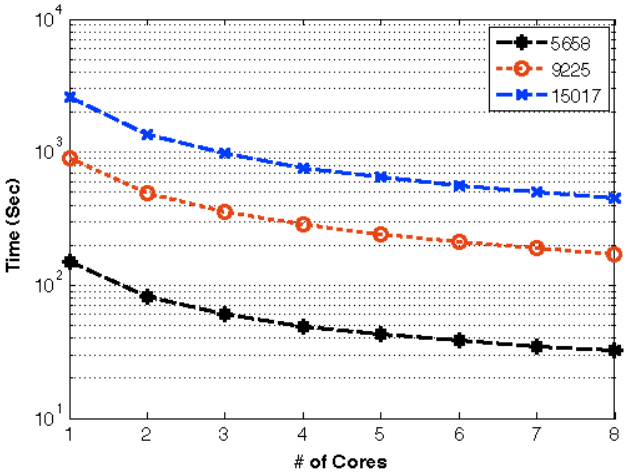

Fig. 2: Time of computation (in sec) is shown for varying number of cores/processors used in a single machine using open-MP, given for three different mesh sizes.

The results using MPI implementation with parallel LAPACK is given in Fig. 3 for four different node sizes. Up to 8 machines were pooled together for processing, allowing us to extend mesh sizes up to 100,000 surface nodes. There was an order of magnitude speed-up in time, which increased with size of the problem.

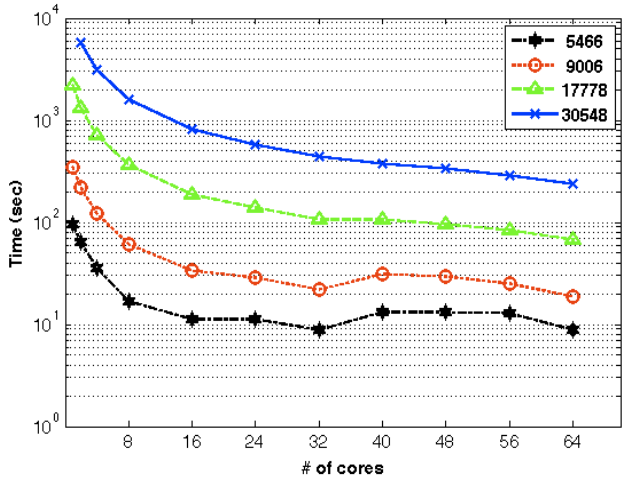

Fig. 3: Time of computation (in sec) is shown for varying number of cores using MPI, given for four different mesh sizes.

IV. DISCUSSION & CONCLUSIONS

BEM is now available as an add-on to open source light diffusion modeling software NIRFAST[13]. Use of parallelization allows us to significantly increase the sizes of problems that can be solved in 3-D image-guided DOT and fluorescence. The use of coupled method allows heterogeneities to be successfully modeled. BEM is currently being extended to image-guided fluorescence using radial basis functions for approximation of the distributed fluorescence source.

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