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A Theoretical Mathematical Modeling of Parkinson's Disease Using Fuzzy Cognitive Maps

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Abstract - **The mathematical model of medical problems is considered. The aim of this paper is to present a new approach in modeling the disease of Parkinson using Fuzzy Cognitive Maps (FCM). Basic theories of FCMs are reviewed and presented. Decision Support Systems (DSS) for Medical problems are considered. The disease of Parkinson is mathematically modeled using Fuzzy Cognitive Maps and three (3) experts. Linguistic variables are proposed and used to describe the correlation among concepts of the FCM. Simulations are performed and very interesting results are obtained and discussed.**

Index terms - fuzzy cognitive map, decision support system, mathematical modeling, parkinson disease.

I. INTRODUCTION

Complexity and large number of data characterize today's Medical Systems while their needs continuously increase. New practices, models, methods and techniques have been emerging as complements to medical procedures and products. Meanwhile, Decision Support Systems (DSS) in the field of Medicine require flexibility, autonomy, intelligence, reliability but above all should be trusted by the physician-doctor. To fulfill all these diverse and difficult requirements, physicians-doctors in close collaboration with scientists and engineers investigate new models and techniques that will integrate and combine known advanced theories and new techniques that will be the core of these sophisticated systems. At the same time, they seek to develop new models and software tools to address complicated issues of medical problems.

A Decision Support System (DSS) is defined as any interactive computer – based support system for making decisions in any complex system, when individuals or a team of people are trying to solve unstructured problems on an uncertain environment.

DSS are especially valuable in situations in which the amount of "scientific data'' is prohibitive for the "human decision maker'' to proceed in solving difficult problems. Advanced DSS can aid human cognitive deficiencies by integrating various methodologies and tools utilizing a number of different information sources in order to reach "acceptable decisions''. The benefits in using DSS are that they increase efficiency, productivity, competitiveness, and offer cost effectiveness and high reliability. This gives business and other "systems'' a comparative advantage over other competitors [1].

The purpose of this paper is to focus on the construction and the use of FCM in modeling a Decision Support System which diagnoses Parkinson's disease. Since the late 1990s, theories of Fuzzy Logic have been used in Medicine and Bioinformatics [9], [10], but not in FCMs. In this paper we present in a simple but illustrative way how useful the FCMs can be in medical problems. In Section 2 the basic theories of Fuzzy Logic in decision making support system are presented, while the decision making support system in Parkinson's disease is described in Section 3. In Section 4, the simulation results are described. The paper concludes in Section 5.

II. BASIC THEORIES OF FUZZY LOGIC IN DECISION MAKING SUPPORT SYSTEM

The method used to develop and construct a Decision Making Support System (DMSS) using FCMs has considerable importance in order to represent the policy decision procedure as accurately as possible. The methodology described here extracts the knowledge from the experts and exploits their experience of the process [2].

The appropriate experts, consisting, in most cases, of interdisciplinary teams, determine the number and kind of concepts that comprise the FCM models of the DMSS. Each expert based on his/ her experience knows the main factors that contribute to the decision; each of these factors is represented by one concept of the FCM. The expert also understands potential influences and interactions between factors themselves or between factors and decisions, thus establishing the corresponding fuzzy degrees of causation between concepts. In this way, expert's knowledge is transformed into a dynamic weighted graph, the DMSS using FCMs. Experts describe the existing relationship between the concepts firstly, as ''negative'' or ''positive'' and secondly, as a degree of influence using a linguistic variable, such as ''low'', ''medium'', ''high'', etc.

More specifically, the causal interrelationships among concepts are declared using the variable Influence which is interpreted as a linguistic variable taking values in the universe of discourse U = [-1, 1]. Its term set T (influence) is suggested to be comprised of nine variables. Using nine linguistic variables, an expert can describe the influence of one concept on another in detail and can discern it between different degrees. The nine variables used here are: T(influence) = {negatively very strong, negatively strong, negatively medium, negatively weak, zero, positively weak, positively medium, positively strong, positively very strong}. The corresponding membership functions for these terms are shown in Figure 1 and they are μ_{nvs}, μ_{ns}, μ_{nm}, μ_{nw}, μ_z, μ_{pw}, μ_{pm}, μ_{ps} and μ_{pvs}.

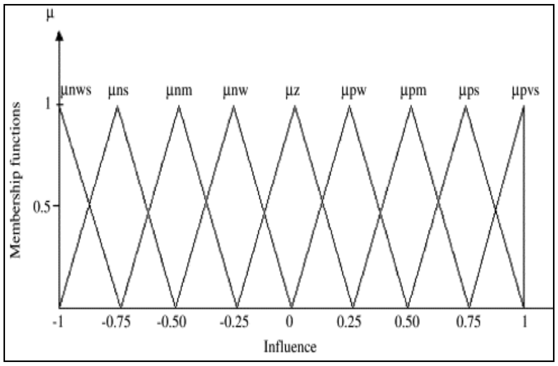

Figure 1. Membership functions of the linguistic variable Influence

Once one expert describes each interconnection as above, then, all the proposed linguistic values for the same interconnection, suggested by experts, are aggregated using the SUM method and an overall linguistic weight is produced, which with the defuzzification method of center of area (COA), is transformed to a numerical weight W_{ji}, belonging to the interval [-1, 1]. A detailed description of the development of FCM model is given in [3].

Generally, the value of each concept at every simulation step is calculated, computing the influence of the interconnected concepts to the specific concept, by applying the following calculation rule [2], [4]:

$$A_i^{(k+1)} = f\left(k_2 A_i^{(k)} + k_1 \sum_{\substack{j \neq i \\ j=1}}^{N} A_j^{(k)} w_{ji}\right) \quad (1)$$

where $A_i^{(k+1)}$ is the value of the concept C_i at the iteration step k+1, $A_i^{(k)}$ is the value of the concept C_j at the iteration step k, W_{jj} is the weight of the interconnection between concepts C_j and C_j and f is the sigmoid function. "k_1" expresses the influence of the interconnected concepts in the configuration of the new value of the concept A_i and k_2 represents the proportion of the contribution of the previous value of the concept in the computation of the new value.

The sigmoid function f belongs to the family of squeezing functions, and the following function is usually used to describe it:

$$f = \frac{1}{1 + e^{-\lambda x}} \quad (2)$$

This is the unipolar sigmoid function, in which $\lambda > 0$ determines the steepness of the continuous function f(x). The following examples show how the FCMs lead to the proposed decision making approach following the experts' knowledge strictly.

III. DECISION MAKING SUPPORT SYSTEM IN PARKINSON'S DISEASE

The method described above will be used in order to diagnose Parkinson's disease. It is considered that $k_1=k_2=1$, $\lambda=1$ and an initial matrix $W_{initial}=[W_{ij}]$, i,j=1,…,N, with $W_{ii}=0$, i=1,…,N, is obtained.

The current Decision Making Analysis model consists of eight concepts. Concept 8 is the decision concept (output), which will show the exact stage of Parkinson's disease. The factor concepts are seven and describe the symptoms. Specifically these concepts are the following:

- C_1:Body Bradykinesia (Slowness of movement)
- C_2: Rigidity (Stiffness of muscles)
- C_3: Posture
- C_4: Movement of upper limbs
- C_5: Gait
- C_6: Tremor
- C_7: Self-care
- C_8: Stage of Parkinson's disease (output)

The weights between concepts are presented in the matrix_1 below:

$$W = \begin{bmatrix} 0 & 0.79 & 0 & -0.42 & 0.75 & 0 & 0.67 & 0.67 \\ 0.79 & 0 & 0 & -0.58 & 0.25 & 0 & 0.25 & 0.25 \\ 0.25 & 0 & 0 & -0.17 & 0.67 & 0 & 0.25 & 0.17 \\ 0.79 & 0.79 & 0.79 & 0 & 0.88 & 0 & 0.25 & 0.25 \\ 0.75 & 0.67 & 0.33 & 0.79 & 0 & -0.75 & 0.75 & 0 \\ 0.17 & 0 & 0 & 0 & 0.75 & 0 & 0.79 & 0.83 \\ 0.83 & 0.88 & 0.75 & 0 & 0.83 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix 1

In order to show how the crisp values of the weights created in Matrix_1, we are going to give a specific example for the calculation of the crisp value of a single weight describing the correlation between node C_1 (bradykinesia) and node C_2 (rigidity). Preferences of three experts on how they define this correlation follow:

1st expert:
If a very small change occurs in value of concept C_1 then a very strong change in value of concept C_2 is caused.
Infer: Influence of C_1 to C_2 is positively very strong.

2nd expert:
If a small change occurs in value of concept C_1 then a strong change in value of concept C_2 is caused.
Infer: Influence of C_1 to C_2 is positively strong.

3rd expert:
If a small change occurs in value of concept C_1 then a strong change in value of concept C_2 is caused.
Infer: Influence of C_1 to C_2 is positively strong.

Figure 2 shows the three linguistic variables which are being proposed.

These linguistic variables (very strong, strong, strong) are aggregated and a total linguistic weight is produced, which is transformed into a crisp value $W_{12}=0.79$ after the CoA defuzzyfication method [5], [6], [7].

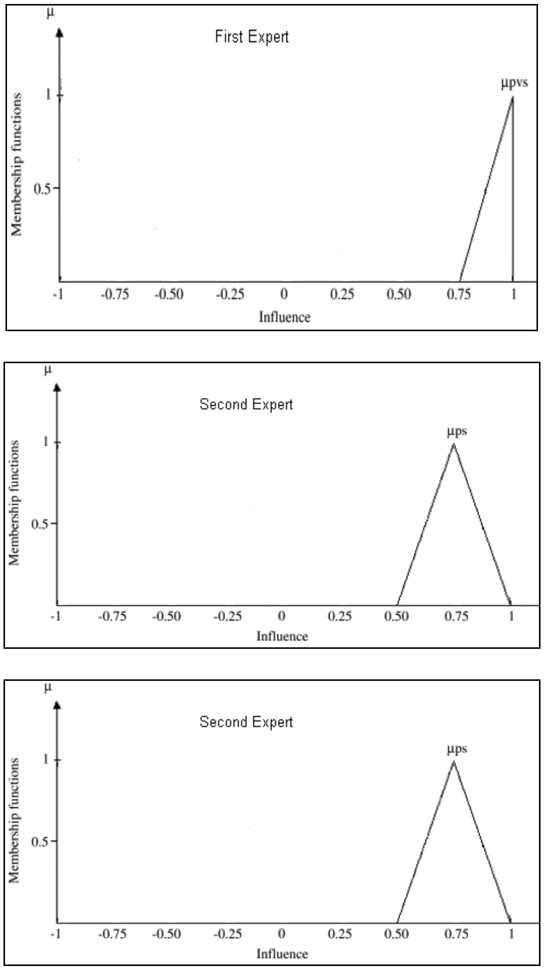

Figure 2. Example of the three linguistic variables proposed by three experts to describe the correlation among two concepts

The initial Fuzzy Cognitive Map with the first values of concepts will be as follows:

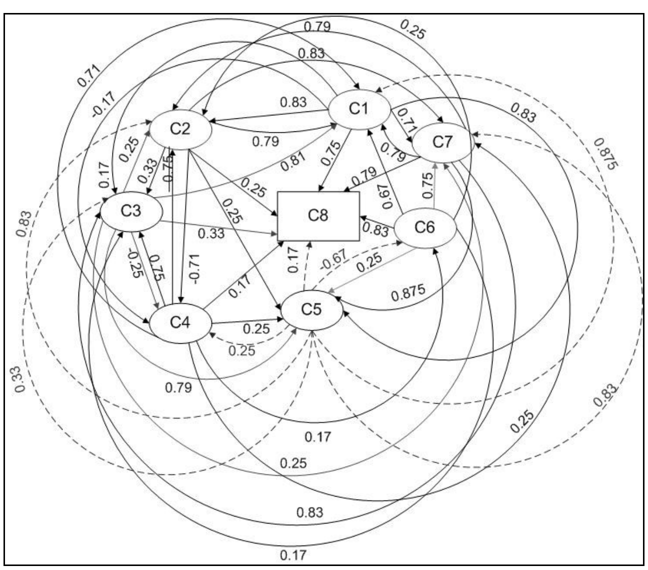

Figure 3. The FCM model

The same procedure was followed for the determination of the remaining weights of the FCM model. A weight matrix $W_{initial}$ which contains the initially proposed weights of all interconnections among the concepts of the FCM model is shown in Matrix_1.

The initial factor-concepts are being defined by physicians after their patients' examination. Every physician will determine the inputs C_1-C_7 with a linguistic variable. This means that these initial values should be transformed in the interval [0, 1] according to the following rules:

If physician decides the patient has no symptoms **then** C_i=0.

If physician decides the patient's symptoms are weak **then** C_i=0.25.

If physician decides the patient's symptoms are medium **then** C_i=0. 5.

If physician decides the patient's symptoms are strong **then** C_i=0.75.

If physician decides the patient's symptoms are very strong **then** C_i=1.

The numerical value of node C_8 (output) depends on the value of inputs (C_1-C_7). Their relationships are described by experts in the Table_1. Consequently vector A= [C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8] will be created.

Each numerical value of the output corresponds to a linguistic variable, which represents the output. The linguistic variables of the output are:

- healthy
- stage_1
- stage_2
- stage_3
- stage_4
- stage_5

TABLE I. NUMERICAL VALUE OF THE OUTPUT, DEFINED BY EXPERTS

C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	**C8 (OUT)**
[0, 0.25]	[0, 0.25]	[0, 0.25]	[0, 0.25]	[0, 0.25]	[0, 0.25]	[0, 0.25]	**0**
[0, 0.5]	[0.25, 0.5]	[0, 1]	[0, 0.5]	[0, 1]	[0.25, 0.5]	[0, 1]	**0.2**
[0.25, 0.5]	[0.5, 0.75]	[0, 1]	[0, 1]	[0.5, 0.75]	[0.25, 0.75]	[0, 1]	**0.4**
[0.5, 0.75]	[0.5, 1]	[0.25, 0.5]	[0, 1]	[0.75, 1]	[0.5, 1]	[0, 1]	**0.6**
[0.75, 1]	[0.75, 1]	[0.5, 0.75]	[0, 1]	[0.75, 1]	[0.75, 1]	[0, 1]	**0.8**
[0.75, 1]	[0.75, 1]	[0, 1]	[0, 1]	[0.75, 1]	[0.75, 1]	[0.75, 1]	**1**

IV. SIMULATION RESULTS AND DISCUSSION

1st CASE

Suppose that the physician decided as initial values of the inputs the following:

TABLE II. INITIAL FACTOR CONCEPTS

C_1	Strong
C_2	Strong
C_3	Medium
C_4	Medium
C_5	Strong
C_6	Very Strong
C_7	Medium

The initial value of the output results from Table_1 and Table_2 and it is C8=1.

We considered initial values for the concepts after COA defuzzyfication method:

$A^{(0)}$= [0.75 0.75 0.5 0.5 0.75 1 0.5 1]

The iterative procedure is being terminated when the values of C_i concepts have no difference between the latest two iterations. Considering λ=1 for the unipolar sigmoid function and after N=9 iteration steps the system reaches an equilibrium point.

The fuzzy rule considered for the calculation of the initial conditions of the output concept C_8 follows:

IF C_1 is S **and** C_2 is S and C_3 is M and C_4 is M **and** C_5 is S **and** C_6 is VS **and** C_7 is M **THEN** C_8 is VS.

According to Table_3 below, it is observed that in the latest two iterations there is no difference between the values of concepts C_i. So after 9 iteration steps, the FCM reaches an equilibrium point where the values do not change any more from their previous ones, that is:

$A^{(9)}$= [0.9827 0.9749 0.9174 0.5517 1.000 0.4177 0.9649 0.9619]

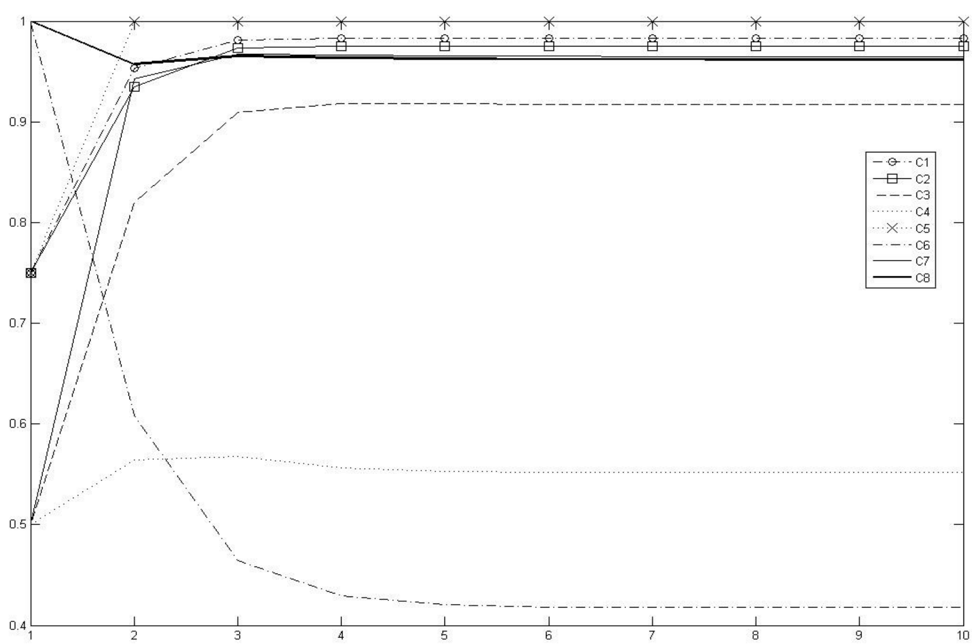

Figure 4. Subsequent values of concepts till convergence

TABLE III. VALUE OF CONCEPTS FOR EACH ITERATION STEP

	C1	C2	C3	C4	C5	C6	C7	C8
1	0.9687	0.9500	0.8412	0.5703	0.9777	0.6179	0.9555	0.9681
2	0.9898	0.9824	0.9283	0.5660	0.9908	0.4684	0.9771	0.9759
3	0.9904	0.9838	0.9356	0.5562	0.9908	0.4251	0.9760	0.9742
4	0.9902	0.9836	0.9355	0.5529	0.9904	0.4135	0.9751	0.9731
5	0.9902	0.9836	0.9352	0.5520	0.9903	0.4105	0.9748	0.9728
6	0.9901	0.9835	0.9352	0.5517	0.9903	0.4098	0.9747	0.9727
7	0.9901	0.9835	0.9351	0.5517	0.9903	0.4096	0.9747	0.9726
8	0.9901	0.9835	0.9351	0.5516	0.9903	0.4095	0.9747	0.9726
9	0.9901	0.9835	0.9351	0.5516	0.9903	0.4095	0.9747	0.9726

Finally for the interpretation of the results, an average only for the output value of the decision concept C_8 is computed according to the following criterion [8]:

$$R(x)=\begin{cases} 0, x \leq 0.5 \\ \dfrac{x-0.5}{0.5}\times 100\%, x > 0.5 \end{cases} \quad (3)$$

This criterion can be modified according to the expert's judgment.

So in our case the calculated value of the decision concept is C_8=0.9619, which following the above criterion in equation (3) corresponds to the 92.4% of the output. Consequently the stage of Parkinson's disease is **stage_5**.

2nd CASE

If we choose three other experts for the same patient, they will decide different weights. In this case we suppose that the final weight matrix (matrix_2), resulting from the three experts as previously, will be the following:

$$W=\begin{bmatrix} 0 & 0.88 & 0 & -0.17 & 0.75 & 0 & 0.79 & 0.67 \\ 0.79 & 0 & 0 & -0.42 & 0.25 & 0 & 0.25 & 0.25 \\ 0.25 & 0 & 0 & -0.17 & 0.67 & 0 & 0.33 & 0.17 \\ 0.79 & 0.88 & 0.79 & 0 & 0.88 & 0 & 0.17 & 0.25 \\ 0.83 & 0.67 & 0.58 & 0.42 & 0 & -0.75 & 0.75 & 0 \\ 0.33 & 0 & 0 & 0 & 0.75 & 0 & 0.79 & 0.83 \\ 0.88 & 0.83 & 0.75 & -0.17 & 0.75 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix 2

Using this matrix and following the methodology above, the calculated value of the decision concept results in C_8=0.9618 and the FCM reaches this equilibrium point after 9 iteration steps. The final vector A will be the following:

$A^{(9)}$= [0.9854 0.9774 0.9334 0.5062 1.000 0.4177 0.9697 0.9618]

By using the above criterion in equation (3) the output corresponds to the 92.3% of the output. Consequently the stage of Parkinson's disease is **stage_5**.

The conclusion is that the stage of the disease is the same in both cases. Therefore, if the weight matrices have small deviations, the output will be an approximately equal number, which will also be the stage number of Parkinson's disease.

V. FUTURE RESEARCH

The interesting results obtained here show new and promising future research areas. It is obvious that theories of FCMs which have not been used in Medical problems can be explored and other health problems similar to Parkinson's disease can be studied further. Interacting with doctors and scientists from the Medical profession develops more suitable FCM models for health problems.

There is a need for software tools for Medical Decision Support Systems using theories of FCMs. In addition learning algorithms, such as Hebbian and non-linear, should be considered to further investigate Medical problems. The challenging research is the development of new suitable software tools. An interesting research topic is the development of recursive dynamic state equations for FCMs and the use of them for mathematical modeling of medical problems.

VI. SUMMARY AND CLOSING RESULTS

In this paper, the mathematical theories and tools of FCM, developed previously, have been used to theoretically model the Parkinson's disease. A previously developed algorithm was used to demonstrate the usefulness of the FCM approach in modeling medical problems. The simulation results using hypothetical data show that the proposed approach can help in further understanding and treating more effectively Parkinson's disease.

In conclusion that Decision Support System could be used both by physicians and hospitals in order to achieve a fast and accurate diagnosis, since after only a few recursive steps the FCM reaches the desired result. Furthermore DSS should be extended to diagnose a wide range of diseases, and also find the treatment that should be followed.

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