A Scheme for X-ray Medical Image Denoising using Sparse Representations

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***Abstract*—This paper addresses the problem of noise removal in X-ray medical images. A novel scheme for image denoising is proposed, by leveraging recent advances in sparse and redundant representations. The noisy X-ray image is decomposed, with respect to an overcomplete dictionary which is either fixed or trained on the noisy image, and it is reconstructed using greedy techniques. The new scheme has been tested with both artificial and real X-ray images and it turns out that it may offer superior denoising results as compared to other existing methods.**

I. Introduction

X-ray computed tomography (CT scan) is a widely used medical imaging modality that produces tomographic images (slices) of the human body. However, exposure to X-ray radiation should be kept as low as possible due to the risks associated with the use of ionized radiation. Consequently, it is of primary importance to reduce the radiation dose during X-ray imaging while keeping image quality high. However, the quality of the reconstructed tomographic images is dependent on exposure and a lower dose leads to reduced Signal to Noise Ratio (SNR) and sharpness which affects medical diagnosis.

Therefore, various methods have been proposed as solutions to the so-called image denoising problem, which in our context consists in reducing noise in the CT image domain. The simplest ones are based on linear filtering using temporal or spatial low-pass filters. The major drawback of these methods is that apart from noise reduction they also cause increased blurriness and hence a reduction of signal information. High-pass filters such as Ramp [1] sharpen the edges of the image but present poor denoising performance. Other approaches include nonlinear filtering [2] , bilateral filtering [3] and wavelet based noise reduction [4] .

In this paper, a different approach for noise reduction based on recent advances in the theory of sparse representations of signals, is proposed. The theory has been widely studied and has had many applications in the field of image processing such as inpainting [5] , compressive sampling [6] and super-resolution [7] . In this work, a denoising technique based on sparse and redundant representations over learned dictionaries [8] is presented and applied on X-ray medical images.

The desired sparse decomposition of the images under study can be done using a greedy pursuit algorithm which finds the sparsest approximate representation of the signal given a specific dictionary. The method is also called sparse-coding. The dictionary may either be a pre-specified linear transform or may result from a training process which adapts its content so as to fit a certain set of example signals. In the latter case, the K-SVD algorithm [9] is adopted for dictionary training whereas the sparse-coding of the signal is performed using the so-called Orthogonal Matching Pursuit (OMP) algorithm.

As it has been shown through extensive simulations, using both artificial and real X-ray images, the proposed scheme offers superior denoising performance as compared to existing methods.

II. Problem Formulation

A. *Image Denoising*

In this work, we consider the fundamental problem of image denoising, which is expressed as the recovery of an image contaminated by additive noise. Let us denote by $\boldsymbol{X} \in \mathbb{R}^{\sqrt{N}\times\sqrt{N}}$ an ideal image which is acquired in the presence of an additive zero-mean white and homogeneous Gaussian noise $\boldsymbol{V} \in \mathbb{R}^{\sqrt{N}\times\sqrt{N}}$, with standard deviation σ. The measured image $\boldsymbol{Y}$ is given by the following expression

$$\boldsymbol{Y} = \boldsymbol{X} + \boldsymbol{V}$$

In general, the X-ray image additive noise is described according to the Poisson distribution. However, for a sufficiently large number of quanta contributing per pixel, the Poisson distribution can be approximated by a Gaussian distribution with the same mean and variance [10] .

B. *Sparse Representation of Images*

Recently, it has been shown that sparse and redundant representation of images may be a highly effective tool for image denoising. More specifically, using an overcomplete *dictionary* matrix $\mathbf{D} \in \mathbb{R}^{N\times K}$, with K atoms (columns) and $K > N$, a signal may be represented, under some assumptions, as a sparse linear combination of these atoms.

Manuscript received July 30, 2013. This research has been supported by CEID, University of Patras, and co-financed by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Aristeia. Investing in knowledge society through the European Social Fund.

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978-1-4799-3163-7/13/$31.00 ©2013 IEEE

Thus, a vectorized image $\boldsymbol{x} \in \mathbb{R}^{N\times 1}$ may be represented by using a small number of atoms of the dictionary (matrix) **D**, i.e. $\boldsymbol{x} = \boldsymbol{D\alpha}$. However, in our case, due to Gaussian noise and modeling errors, the sparse representation cannot be exact, therefore we have $\boldsymbol{x} \approx \boldsymbol{D\alpha}$, satisfying $\|\boldsymbol{y} - \boldsymbol{D\alpha}\|_2^2 \leq \varepsilon$, where ε is the error tolerance. In order to recover a sparse representation $\boldsymbol{\alpha}$ from the noisy image **y** we must solve the following constrained optimization problem

$$\widehat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \text{ subject to } \|\mathbf{D}\boldsymbol{\alpha} - \mathbf{y}\|_2^2 \leq \varepsilon \quad (2)$$

where $\|\bullet\|_0$ is the ℓ_0 pseudo-norm which counts the non-zero elements of $\boldsymbol{\alpha}$. The approximation of the clean image can then be computed by solving $\widehat{\boldsymbol{x}} = \boldsymbol{D}\widehat{\boldsymbol{\alpha}}$. Since an exact determination of the sparsest representation is an NP-hard problem, several alternative methods that lead to approximate solutions have been considered in literature (e.g.,[11] , [12]).

III. Proposed Scheme

In this section, we propose an efficient denoising scheme for X-ray images, leveraging sparse and redundant representations. Sparse representation may be achieved through the solution of (2) using a greedy algorithm. Furthermore, the involved dictionary **D** might either be chosen a priori, or it should be designed by adapting its content to fit a given set of images. In the first case, we employ the OMP algorithm [13] , in order to approximate the solution of (2). In the second case, the K-SVD methodology [9] is adopted, which operates in two stages, first the dictionary is constructed by training and then the clean image is estimated using OMP. In both cases, the algorithms have been properly adapted in order to meet the X-ray CT application requirements, which are described in the experiments section.

A. *Static Dictionary*

In the case where the dictionary is static and a priori known, we seek for an approximation to the problem (2), which can be written in the following equivalent form

$$\min_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\| \text{subject to } \|\boldsymbol{\alpha}\|_0 \leq \mathrm{L} \quad (3)$$

where L is a constraint on the number of dictionary atoms that participate in the sparse representation of the image. Hence, we use OMP for the solution of (3), an algorithm that given a specific dictionary aims at finding a sparse representation of the input image. The columns of the dictionary matrix that participate in the linear combination are determined iteratively in a greedy fashion. At each iteration, the dictionary column that is most strongly correlated with the remaining part of the image is chosen. Given a dictionary **D** and a noisy image **y**, the corresponding sparse vector **α** is found through the steps of Table 1. The stopping rule can either be based on the sparsity of vector **α** or on the reconstruction error $\|\boldsymbol{y} - \boldsymbol{D\alpha}\|_2^2$ **.**

B. *Adaptive Dictionary*

The goal of dictionary training is to find a matrix that best describes the content of a set of training images and leads to the best representation for each example image under strict sparsity constraints. In this work, the algorithm for designing adaptive overcomplete dictionaries is based on the K-SVD methodology (Table 2). The K-SVD algorithm is an iterative method and consists of two stages, the sparse coding stage and the dictionary update stage. At the first stage, the dictionary **D** is assumed fixed and the sparse representations of the training signals are calculated by using the OMP. The sparse vector sought is the one that minimizes the representation error and maintains the sparsity constraint.

<u>Initialization</u>: Choose a static dictionary $\boldsymbol{D}$
To find $\widehat{\boldsymbol{\alpha}}$ repeat until convergence:
1. Select the atom with maximal projection on the residual
2. Update $\boldsymbol{\alpha}^i = \operatorname{argmin}_{\boldsymbol{\alpha}} \|\mathbf{y} - \mathbf{D}\boldsymbol{\alpha}\|_2$
3. Calculate new approximation of the image $\hat{\mathbf{x}}^i = \mathbf{D}\boldsymbol{\alpha}^i$
4. Update the residual $\mathbf{r}^i = \mathbf{x} - \hat{\mathbf{x}}^i$.

Compute clean image $\widehat{\boldsymbol{x}} = \boldsymbol{D}\widehat{\boldsymbol{\alpha}}$

Table 1: Static Dictionary Scheme

<u>Initialization</u>: An initial dictionary $D^{(0)} \in \mathbb{R}^{nxK}$ with ℓ_2 normalized columns is set.
Repeat until convergence:
- **Sparse coding stage**: the representation vectors x_i for each example y_i are computed using OMP for the approximation of (3)
- **Dictionary update stage** : each column $d_k, k = 1, 2, \dots K$ in $D^{(J-1)}$, gets updated :
  - Define the training images $\omega_k = \{i | 1 \leq i \leq N, x_T^k(i) \neq 0\}$.
  - The overall representation error matrix, E_k, is computed :
    $$E_k = Y - \sum_{j \neq k} d_j x_T^j$$
  - By clearing off the columns of E_k that do not correspond to ω_k, we obtain E_k^R.
  - Final step is the SVD decomposition, i.e., $E_k^R = U\Delta V^T$. The updated dictionary column $\tilde{d}_k$ is the first column of U and the updated coefficient vector x_R^k is the first column of V multiplied by $\Delta(1,1)$.

Table 2: Adaptive Dictionary Scheme

At dictionary update stage, **D** gets tuned to the new estimate and each dictionary atom is updated iteratively. This methodology targets at the solution of the following constrained optimization problem

$$\min_{\mathbf{D},\mathbf{A}} \{ \|\mathbf{Y} - \mathbf{DA}\|_F^2 \} \text{ subject to } \forall\, i, \|\boldsymbol{\alpha}_i\|_0 \leq \mathrm{L} \quad (4)$$

where $\|.\|_F$ stands for the Frobenius norm and **A** is the matrix containing the sparse representation of each column $\boldsymbol{\alpha}_i$.

The training process precedes the actual denoising steps, adding computational overhead that is defined by the sparse coding stage. The sparse coding stage utilizes the OMP algorithm that is also the core of the denoising phase. Consequently, training and denoising can be considered processes of equal computational burden.

IV. Experimental Setup

In this work, experiments were conducted using two different sets of X-ray images: the first one with artificial images, and the second with real and experimental data. The set of artificial images was produced using the XrayImagingSimulator [14] . The images are acquired using a simple breast phantom irradiated with $1x10^6$ photons at 20KeV. The test image to denoise, is an ideal projection image taken from the same breast phantom corrupted by artificially added noise of Gaussian distribution.

In the second part of this study, having determined the conditions that lead to the best results, our denoising scheme is tested on real X-ray CT images and on a set of 9 experimental projection images of a breast phantom with highly heterogeneous background acquired at ELETTRA Synchrotron facilities using a monochromatic beam of 17keV for an arc length of 32° [15] . The denoised projection images were then reconstructed using Multiple Projection Algorithm (MPA) [16] and the feature of interest is a group of 7 calcifications on the left upper corner of the image.

As this noise reduction scheme assumes knowledge of noise standard deviation, $\sigma$is estimated before running the tests by calculating the standard deviation of the gray values of background pixels.

V. Experimental Results

In this section, we compare the performance of the proposed scheme both visually and with respect to Peak Signal to Noise Ratio (PSNR) of the denoised image. The focus is on four different dictionary choices: a fixed Discrete Cosine Transform(DCT) dictionary, a general K-SVD dictionary trained on clean natural images, a specialized K-SVD dictionary trained on clean simulated X-ray projection images and a K-SVD dictionary trained on the corrupted image to be denoised. Their performance is tested in the presence of noise of varying standard deviation that corresponds to different dose levels, keeping in mind that the dose is inversely proportional to the noise level.

To train the general K-SVD dictionary, a training matrix is constructed with columns being the patches taken from ten noise free natural images. The images were of size 256x256, and their content was unrelated to that of the X-ray images under study. When the selected patch-size is 8x8 pixels, the resulting number of patches is 10240. The training matrix for the specialized K-SVD dictionary is constructed in a similar way using ten artificial noise free X-ray simulated projection images. Initially, we investigate the performance of the four aforementioned dictionaries. Figs. 1 and 2 reveal that a trained dictionary gives better results than a fixed one, especially when the training image set is of the same type as the image to be denoised. However, the optimal performance is achieved when the corrupted image is also used for dictionary training.

As expected, the performance is decreased as the noise standard deviation increases, or equivalently as the X-ray radiation dose increases. Regarding the size of the patch, the sizes 8x8 and 10x10 are shown to be ideal. Dictionary size seems to have little importance and as shown in Fig. 2 (b), using more than 6 atoms to represent an image patch does not affect the result.

With this knowledge, the next tests were implemented using only dictionaries trained on the noisy images to be cleaned. The selected dictionary consisted of 256 atoms and the size of the patches was 8x8 pixels.

Figure 3 depicts the results of the comparison between K-SVD denoising, Ramp filtering and Median filtering, in terms of PSNR of the denoised image versus the standard deviation of noise. As it can be seen the proposed method exhibits considerably better performance as compared to the other methods. In Figure 4, an example of denoising on a real X-ray CT image is demonstrated. A significant increase in Contrast to Noise Ratio (CNR) can be easily seen. Finally in Figure 5, except of noise reduction, an improved appearance of the calcifications and the breast anatomy in general can be observed.

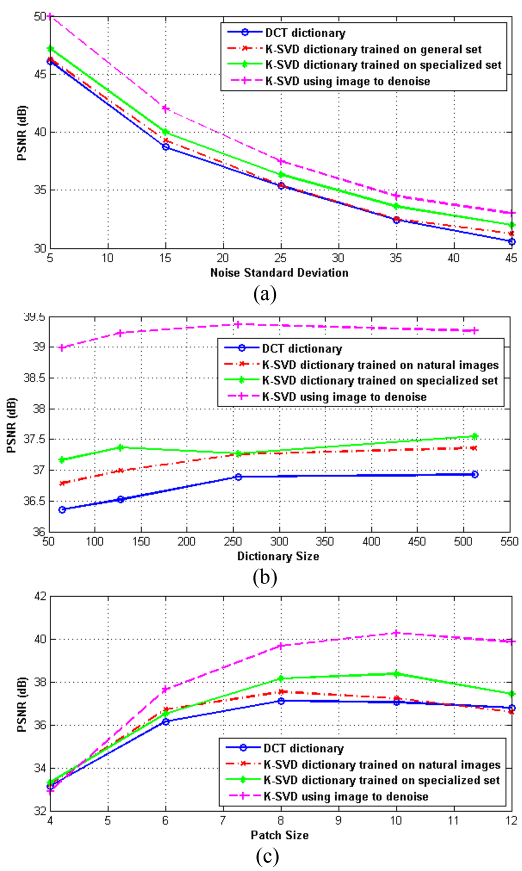

Fig. 1 Comparison of different dictionaries in terms of PSNR of the denoised image versus: (a) noise standard deviation, (b) the dictionary size: (c) the size of image patch.

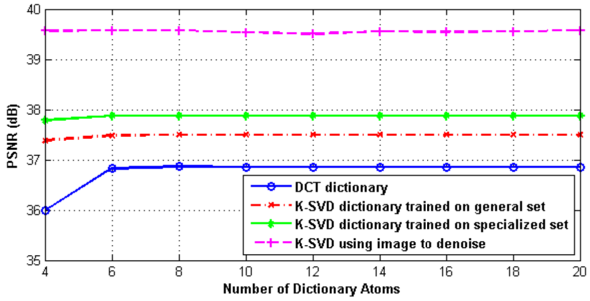

Fig. 2 Comparison of different dictionaries in terms of PSNR of the denoised image versus the number of nonzero elements of the representation vector.

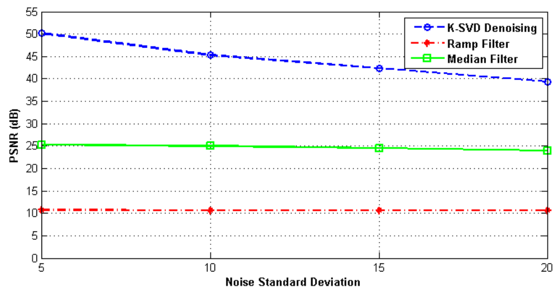

Fig. 3Comparison of K-SVD denoising, Ramp filter and Median filter in terms of PSNR of the denoised image versus noise standard deviation.

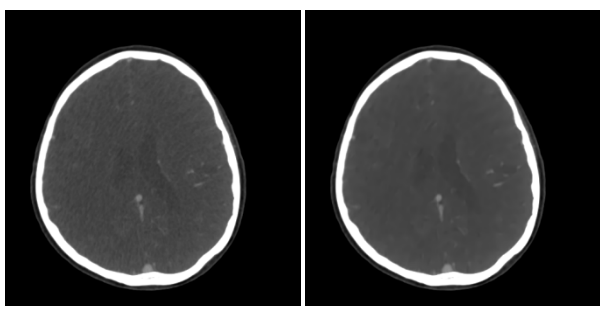

Fig. 4 Left: original noisy image, CNR=9.78. Right: denoised image, CNR = 16.82.

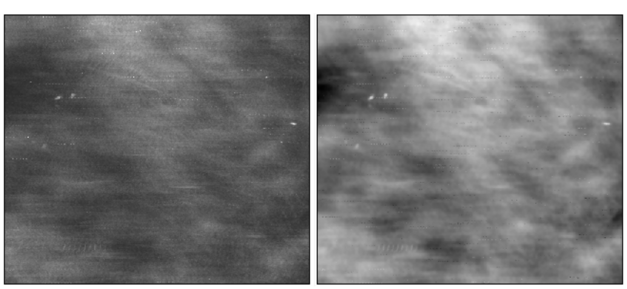

Fig. 5 Left: noisy reconstruction. Right: reconstruction after K-SVD denoising on projection images.

VI. Conclusions And Future Work

In this work, the problem of removing noise from CT X-ray images was addressed. A method based on sparse and redundant representations of images over learned dictionaries is proposed and the issue of suitably determining the involved parameters for best denoising performance was investigated. It was shown that the denoised images exhibit a noticeable improvement in quality as compared to existing methods. Future investigations include in depth comparison of the presented scheme with state-of-the-art noise removal methods used in CT imaging and performance study with different noise models.

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