SIFEM Project: Finite Element Modeling of the Cochlea

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Abstract— **The cochlea is a very interesting part of the body. There are several investigations of experiments on the real cochlea and mathematical models. The cochlea works on the basis of a vibrating system. SIFEM project focuses on the development the multi-scale modelling of the inner-ear with regard to the sensorineural hearing loss. In this study we focused on the finite element model of the cochlea. The first approximation is straight box model where both domain basilar membrane and surrounding fluid are modeled. Fluid-structure interaction problem was implemented. The basilar membrane was modeled as structural plate with 3D brick finite element and fluid domain around the basilar membrane was modeled as full 3D Navier-Stokes equations. ALE formulation was employed for fluid domain and mesh moving algorithm for motion of the membrane and fluid mesh. The results for different frequencies for 3D box and spiral model are presented. It can be observed that viscous fluid allow a sharper response of the membrane, because the viscous fluid would quickly damp out the vibratory motion.**

I. INTRODUCTION

The cochlea is the part of the inner ear where acoustic signals are transformed into neural pulses and then they are signaled to the brain. The precise nature of the mechanisms responsible for the extreme sensitivity, sharp frequency selectivity and the wide dynamic range of the cochlea still remain unknown.

The normal function of the cochlea requires a full integration of mechanical, electrical, and chemical effects on the milli-, micro-, and nanometer scales. There are several papers which include details of the anatomy, for example Pickles [1] and Gulick et al [2]. A summary of analysis and data related to the macromechanical aspect up to 1982 is given by Steele [3], and surveys specifically on the cochlea are by Dallos [4], Ruggero [5], and Nobili et al [6].

In this study we developed a full three-dimensional model for simulation of the cochlea mechanics which include fluid–structure interactions, non-linear Navier–Stokes equations of viscous fluid dynamics and realistic description of the elastic structures. The paper is organized as follows. Section II is a short introduction to cochlear mechanics and general methodology of the fluid-structure interaction method. The method has been extensively tested for different models of the elastic boundary and coupled with the fluid domains. Section III describes some initial results for 3D straight box spiral model for displacement of the basilar membrane and fluid velocity distribution for two-chamber fluid domain. Finally some discussion and conclusions remarks are given.

Manuscript received July 31, 2013. This work was supported in part by grants from Serbian Ministry of Education and Science III41007, ON174028 and FP7 ICT SIFEM 600933.

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II. METHODS

A. Cochlear mechanics

The cochlea is a small snail-shell-like cavity in the temporal bone, which has two openings, the oval window and the round window. The cavity is filled with fluid and is sealed by two elastic membranes that cover the windows. External sounds set the ear drum in motion, which is conveyed to the inner ear by the ossicles, three small bones of the middle ear, the malleus, incus and stapes. The piston-like motion of the stapes against the oval window displaces the fluid of the cochlea, so generating traveling waves that propagate along the basilar membrane. Macro-mechanical system of the cochlea is described by the Navier–Stokes equations of incompressible fluid mechanics coupled with equations modeling the elastic properties of the basilar membrane and the membranes of the oval and the round windows. The displacements of the basilar membrane are extremely small (on a nanometer scale), which means that the system works in a linear regime.

The cochlea model can be presented for simplicity into a single membrane with properties similar to that of the basilar membrane. The properties of the basilar membrane vary along its length but across its length, it is assumed that they are very much similar. The cochlea is spiral-shaped and we made two models: straight box as the first approximation and full 3D spiral.

B. Fluid model

The three-dimensional flow of a viscous incompressible fluid considered here, is governed by the Navier-Stokes equations and continuity equation that can be written as

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\mu\left(\frac{\partial^2 v_i}{\partial x_j\partial x_j}+\frac{\partial^2 v_j}{\partial x_j\partial x_i}\right) \quad (1)$$

$$\frac{\partial v_i}{\partial x_i}=0 \quad (2)$$

where v_i is the blood velocity in direction x_i, ρ is the fluid density, p is pressure, μ is the dynamic viscosity; and summation is assumed on the repeated (dummy) indices, i,j=1,2,3. The first equation represents balance of linear momentum, while the equation (2) expresses incompressibility condition.

Each waveform of flow was discretized into 1000 uniformly spaced time steps. In the analysis, it was considered that the convergence was reached when the maximum absolute change in the nondimensional velocity between the respective times in two adjacent cycles was less than 10^{-3}.

The code was validated using the analytical solution for shear stress and velocities through curve tube [7]. A penalty formulation was used in our solver [7].

The incremental-iterative form of the equations for a time step and equilibrium iteration "i" are:

$$\begin{bmatrix} \frac{1}{\Delta t}\mathbf{M}_v + {}^{t+\Delta t}\mathbf{K}_{vv}^{(i-1)} + {}^{t+\Delta t}\mathbf{K}_{\mu v}^{(i-1)} + {}^{t+\Delta t}\mathbf{J}_{vv}^{(i-1)} & \mathbf{K}_{vp} \\ \mathbf{K}_{vp}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{v}^{(i)} \\ \Delta\mathbf{p}^{(i)} \end{Bmatrix} = \begin{Bmatrix} {}^{t+\Delta t}\mathbf{F}_v^{(i-1)} \\ {}^{t+\Delta t}\mathbf{F}_p^{(i-1)} \end{Bmatrix} \quad (3)$$

The left upper index "t+Δt" denotes that the quantities are evaluated at the end of time step. The matrix $\mathbf{M_v}$ is mass matrix, $\mathbf{K_{vv}}$ and $\mathbf{J_{vv}}$ are convective matrices, $\mathbf{K_{\mu v}}$ is the viscous matrix, $\mathbf{K_{vp}}$ is the pressure matrix, and $\mathbf{F_v}$ and $\mathbf{F_p}$ are forcing vectors. The pressure is eliminated at the element level through the static condensation. For the penalty formulation, we define the incompressibility constraint in the following manner:

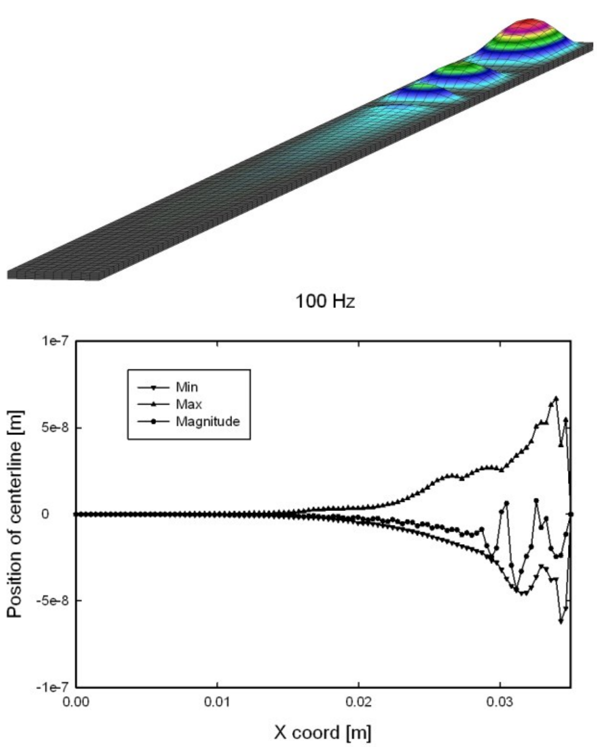

Fig. 1. Basilar membrane displacements distribution (top) and centerline position along distance from base (bottom) with frequency of 100 Hz

$$div\mathbf{v} + \frac{p}{\lambda} = 0 \quad (4)$$

where λ is a relatively large positive scalar so that p/λ is a small number (practically zero).

The incremental-iterative form of the equilibrium equations are

$$\left(\frac{1}{\Delta t}\mathbf{M}_v + {}^{t+\Delta t}\mathbf{K}_{vv}^{(i-1)} + {}^{t+\Delta t}\mathbf{K}_{\mu v}^{(i-1)} + {}^{t+\Delta t}\hat{\mathbf{K}}_{\mu v}^{(i-1)} + {}^{t+\Delta t}\mathbf{J}_{vv}^{(i-1)} + \mathbf{K}_{\lambda v} \right) \Delta\mathbf{v}^{(i)} = {}^{t+\Delta t}\hat{\mathbf{F}}_v^{(i-1)} \quad (5)$$

where the matrices and vectors are given in [8].

C. *Fluid-structure interaction algorithm*

For the complex fluid-structure interaction problem of cochlea with two chamber fluid domains and the basilar membrane deformation, we implemented the loose coupling approach [9]. The overall strategy adopted here consists of the following steps:

a) For the current geometry of the fluid domain, determine fluid flow with use of the ALE formulation [8]. Wall velocities at the common fluid – membrane surface are taken as the boundary condition for the fluid.

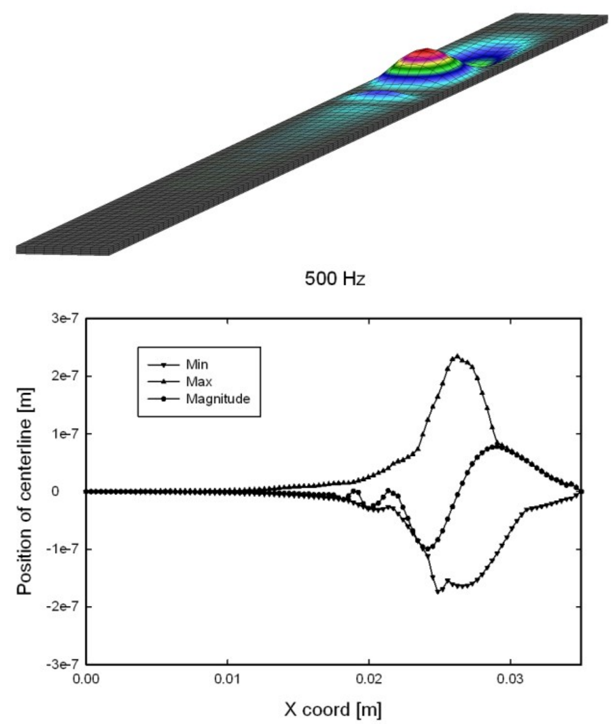

Fig. 2. Basilar membrane displacements distribution (top) and centerline position along distance from base (bottom) with frequency of 500 Hz

b) Calculate the loads, arising from the fluid, which act on the membrane.

c) Determine deformation of the membrane taking the current loads from the fluid domain.

d) Check for the overall convergence which includes fluid and solid. If convergence is reached, go to the next time step. Otherwise go to step a).

The fluid domain geometry and velocities at the common solid-fluid boundary for the new calculation of the fluid flow are updated [9]. In case of large wall displacements, the FE mesh for the fluid flow domain is updated. Go to step a).

III. Results

The basilar membrane displacements distribution and centerline position along distance from base with frequency of 100 Hz have been shown in the Fig. 1 Upper part of the figure represents 3D displacements of the basilar membrane and lower part of the figure shows minimum, maximum and magnitude displacement for the current frequency . Similar results for frequency of 500 and 1000 Hz are presented in Figs. 2 and 3. The results for velocity of the fluid domain for the straight box model as well as 3D spiral model are presented in Fig. 4. The straight box model is using for simplification of the problem and investigation of the parameter as nonlinear elasticity of the basilar membrane and acoustic wave propagation through the fluid. Three-dimensional spiral model gives more anatomical results for cochlear biomechanics problem. Boundary conditions for both models are prescribed velocity on the upper fluid part of the basilar membrane, fixed part of the basilar membrane and free connection between two fluid parts. Prescribed velocity boundary conditions are induces by the input frequency which was indicated in the presented results.

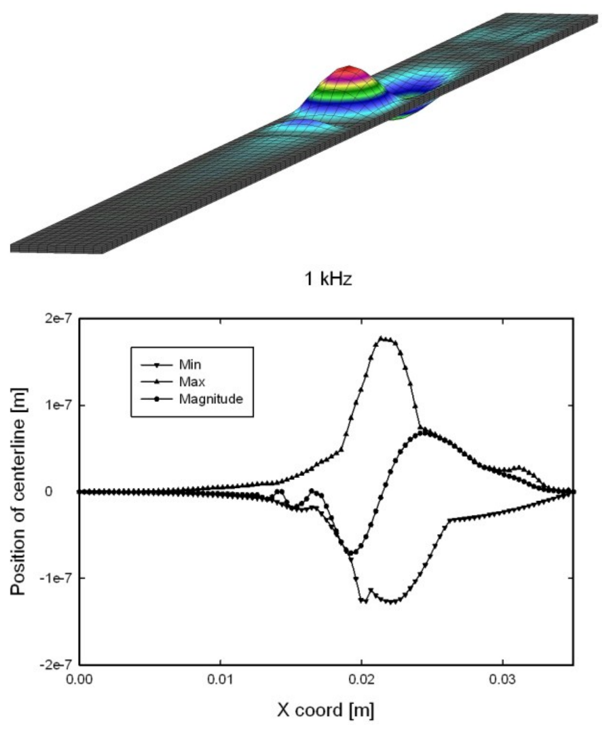

Fig. 3. Basilar membrane displacements distribution (top) and centerline position along distance from base (bottom) with frequency of 1000 Hz

IV. Discussion and Conclusions

We have constructed a comprehensive three-dimensional computational model of the passive cochlea using the loose coupling fluid-structure interaction. The traveling wave box model described in this paper presents the most important properties of the cochlear macro-mechanics. Three-dimensional simplified box and spiral model are presented. Displacement distribution of the basilar membrane and fluid velocity distribution for two-chamber fluid domain with different frequencies 100 Hz, 500 Hz and 1000 Hz are presented.

During SIFEM project we will compare our simulation results with the available experimental and modeling data. We believe our results demonstrate the promise of large scale computational modeling approach to the study of cochlear mechanics.

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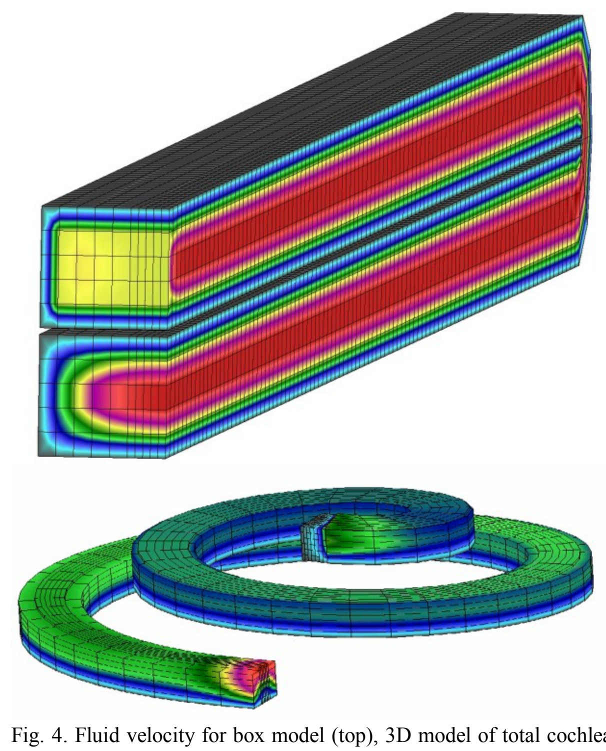

Fig. 4. Fluid velocity for box model (top), 3D model of total cochlea (bottom)

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