EEG Identification of a localized 1-D neuronal excitation

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***Abstract*— Albaneze and Monk [1] have demonstrated that it is impossible to identify the three-dimensional support of any primary current living within a conducting medium, from electromagnetic measurements outside the conductor. However, this is not true if the primary current is supported in a subset of dimensionality less than three. In the present report, we demonstrate the truth of this statement by constructing an analytic algorithm that identifies the location, the orientation, and the size of a localized linear distribution of current dipoles within the brain, from a complete knowledge of the electric potential recorded by an electroencephalographer on the surface of the head.**

I. INTRODUCTION

Electroencephalography (EEG) and Magnetoencephalography (MEG) are the two brain imaging modalities which have the necessary temporal resolution ($10^{-3}\,\mathrm{sec}$) for the study of the functional brain [6], [8]. Since the brain consists of conductive material, any primary neuronal excitation in the brain generates a secondary inductive current within the whole brain tissue. In the framework of the quasi-static theory of Electromagnetism [7], [9] these two currents give rise to an electric potential and a magnetic flux density field. The calculation of the values of the electric potential on the surface of the head forms the forward problem of EEG, while the calculation of the magnetic flux density a few centimeter outside the head forms the forward problem of MEG. The inverse EEG problem seeks to identify the neuronal current within the brain from the knowledge of the electric potential on the surface of the head. The corresponding inverse MEG problem seeks this neuronal current from the knowledge of the magnetic flux 3 to 5 centimeters outside the head, where the SQUID [8] measurements are recorded.

As far as the question of uniqueness of the solutions of these two inverse problem is concerned, that is the characterization of the class of currents that provide identical eclectic potentials on the head, and identical magnetic fluxes outside the head, the ultimate results have been obtain recently [3],[5]. The definitive result states that neither the EEG nor the MEG measurements can recover completely the primary neuronal current, and therefore no uniqueness for the inverse problems exists. An interesting result was obtained along this line in [1]. It states that, if the neuronal current is localized in a genuine three dimensional subset of the brain tissue, a hypothesis which is almost always true, then, it is impossible to recover even the support of the neuronal current either from EEG or from MEG measurements. This impossibility reflects the fact that the current and the relative generated fields are governed by adjoint operators. In fact, this result was also demonstrated in [4], where it is assumed that the primary current is supported in a small sphere centered at the point $\mathbf{r_0}$ with radius ε, and proved that although it is feasible to recover the location of the center of the sphere, it is impossible to obtain its radius since in the process of the necessary analytic calculations the radius disappears from the controlling equations. However, this impossibility is not true if the support of the primary current lives in a subset with dimensionality lower than three. The goal of this report is to demonstrate a simple one dimensional case where the location, the orientation, and the size of a small linear distribution of current dipoles can be recovered from a complete knowledge of the EEG data.

The work is organized as follows. Section 2 involves the statement of the particular EEG problem. Section 3 provides the results of a large amount of analytic calculations needed to express the electric potential, up the degree three, in closed analytic forms. The corresponding inverse problem is postulated in algebraic form in Section 4 and it is shown that all the basic characteristics of the current distribution are indeed recoverable.

II. MATHEMATICAL FORMULATION OF THE EEG PROBLEM

Let us assume that the brain-head system is modeled geometrically as a sphere of radius α, and physically as a homogeneous conductor with conductivity σ. Let denote by $\boldsymbol{J}^p$ the primary neuronal current within the brain, which is actually represented as a discrete or continuous distribution of current dipoles with specific dipole moments. The electric potential u^-, generated in the space $r < \alpha$, solves the interior Newmann boundary value problem

$$\sigma \Delta u^-(\boldsymbol{r}) = \nabla \cdot \boldsymbol{J}^p(\boldsymbol{r}),\ r < \alpha \quad (1)$$

$$\partial_n u^-(\boldsymbol{r}) = 0,\ r = \alpha \quad (2)$$

and the electric potential u^+, generated in the space $r > \alpha$, solves the exterior Dirichlet problem

$$\sigma \Delta u^+(\boldsymbol{r}) = 0,\ r > \alpha \quad (3)$$

$$u^+(\boldsymbol{r}) = u^-(\boldsymbol{r}),\ r = \alpha \quad (4)$$

$$u^+(\boldsymbol{r}) = O(1/r^2),\ r \to +\infty. \quad (5)$$

Suppose now that a neuronal current is supported on a line segment of lenght $2L$ with center at the point $\boldsymbol{r}_0 =$

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(x_{01}, x_{02}, x_{03}) and orientation along the direction $\hat{\boldsymbol{\alpha}} = (\alpha_1, \alpha_2, \alpha_3)$.

We know that [2], if the primary current is given by a single dipole at $\boldsymbol{r}_0$ having the moment $\boldsymbol{Q}$, then the solution of problem (3)-(5) is given by

$$u^+(\boldsymbol{r}) = \frac{1}{4\pi\sigma}\boldsymbol{Q}\cdot\nabla_{\boldsymbol{r}_0}\sum_{n=1}^{\infty}\frac{2n+1}{n}\frac{r_0^n}{r^{n+1}}P_n(\hat{\boldsymbol{r}}\cdot\widehat{\boldsymbol{r}_0}),\ r>\alpha \quad (6)$$

where P_n denotes the Legendre polynomial of degree n. Because of linearity the corresponding solution for the above case where the currents is distributed over the line segment, all we need to do is to replace the isolated dipole ($\boldsymbol{r}_0$,$\boldsymbol{Q}$) in (6) with the distributed current ($\boldsymbol{r}'$,$\boldsymbol{J}^p(\boldsymbol{r}')$) and integrate over the prescribed line segment

$$\boldsymbol{r}'(t) \quad = \quad \boldsymbol{r}_0 + t\hat{\boldsymbol{\alpha}},\ t\in[-L,L]. \qquad (7)$$

For a localized current with a small L, we can represent $\boldsymbol{J}^p$ by the linear approximation of its Taylor expansion around the center point $\boldsymbol{r}_0$, that is by

$$\begin{aligned}\boldsymbol{J}^p(\boldsymbol{r}') &\approx \boldsymbol{J}^p(\boldsymbol{r}_0) + t\frac{d\boldsymbol{r}'(\boldsymbol{r}_0)}{\mathrm{d}t}\cdot\nabla\boldsymbol{J}^p(\boldsymbol{r}_0)\\ &= \boldsymbol{Q} + t\boldsymbol{l} \qquad (8)\end{aligned}$$

where the vector $\boldsymbol{Q}$ represents the average value of the moments and the vector $\boldsymbol{l}$ represents the average directional derivative of the current along the direction $\hat{\boldsymbol{\alpha}}$.

III. Solution of the forward problem

Extremely long and tedious calculation of integrals, based on the terms $n = 1, 2, 3$ of the expansion (6), are needed to obtain the closed form expression in the general case. In fact, performing all the indicated differentiations and integrations with the terms $n = 1, 2, 3$ of the expansion (6), in the generic case, we arrive at the expression

$$u^+(\boldsymbol{r}) = \frac{1}{4\pi\sigma}\left[\frac{H_1(\boldsymbol{r})}{r^3} + \frac{H_2(\boldsymbol{r})}{r^5} + \frac{H_3(\boldsymbol{r})}{r^7}\right] + O\left(\frac{1}{r^5}\right) \quad (9)$$

where H_i , $i = 1, 2, 3$ are given by

$$H_1 \quad = \quad 6L\,(\boldsymbol{r}\cdot\boldsymbol{Q}) \qquad (10)$$

$$H_2 \quad = \quad \frac{5}{2}\left(3\boldsymbol{r}\otimes\boldsymbol{r} - r^2\tilde{\mathbb{I}}\right) : \left(2L\boldsymbol{r}_0\otimes\boldsymbol{Q} + \frac{2}{3}L^3\hat{\boldsymbol{\alpha}}\otimes\boldsymbol{l}\right) \quad (11)$$

$$\begin{aligned}H_3 \quad = \quad & 35L(\boldsymbol{r}_0\cdot\boldsymbol{r})^2(\boldsymbol{r}\cdot\boldsymbol{Q}) + \frac{35}{3}L^3(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{r})^2(\boldsymbol{r}\cdot\boldsymbol{Q})\\ + \quad & \frac{70}{3}L^3(\boldsymbol{r}_0\cdot\boldsymbol{r})(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{r})(\boldsymbol{l}\cdot\boldsymbol{r}) - 14Lr^2(\boldsymbol{r}_0\cdot\boldsymbol{r})(\boldsymbol{r}_0\cdot\boldsymbol{Q})\\ - \quad & \frac{14}{3}L^3r^2(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{r})(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{Q}) - \frac{14}{3}L^3r^2(\boldsymbol{r}_0\cdot\boldsymbol{r})(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{l})\\ - \quad & 7Lr^2r_0^2(\boldsymbol{r}\cdot\boldsymbol{Q}) - \frac{14}{3}L^3r^2(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{r})(\boldsymbol{r}_0\cdot\boldsymbol{l})\\ - \quad & \frac{7}{3}L^3r^2(\boldsymbol{r}\cdot\boldsymbol{Q}) - \frac{14}{3}L^3r^2(\boldsymbol{l}\cdot\boldsymbol{r})(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{r_0}) \qquad (12)\end{aligned}$$

On the other hand, the known exterior field, which comes from a best fitting of the obtained EEG measurements, has the following Taylor expansion in Cartesian form

$$\begin{aligned}u^+(\boldsymbol{r}) \quad = \quad & \frac{1}{4\pi\sigma}\left[\frac{A_1x_1 + A_2x_2 + A_3x_3}{r^3}\right.\\ & + \frac{B_1x_1^2 + B_2x_2^2 + B_3x_3^2}{r^5}\\ & + \frac{B_{12}x_1x_2 + B_{23}x_2x_3 + B_{31}x_3x_1}{r^5}\\ & + \frac{\Gamma_1x_1^3 + \Gamma_2x_2^3 + \Gamma_3x_3^3}{r^7}\\ & + \frac{\Gamma_{12}x_1^2x_2 + \Gamma_{21}x_2^2x_1 + \Gamma_{23}x_2^2x_3}{r^7}\\ & + \frac{\Gamma_{32}x_3^2x_2 + \Gamma_{31}x_3^2x_1 + \Gamma_{13}x_1^2x_3}{r^7}\\ & \left.+ \frac{\Gamma_{123}x_1x_2x_3}{r^7}\right] + O\left(\frac{1}{r^5}\right), r\to\infty \quad (13)\end{aligned}$$

where the coefficients A, $B$and Γ are known expressions of the 13 variables $Q_1, Q_2, Q_3, l_1, l_2, l_3, r_{01}, r_{02}, r_{03}, \hat{\boldsymbol{\alpha_1}}, \hat{\boldsymbol{\alpha_2}}, \hat{\boldsymbol{\alpha_3}},$ and L. Since the function u^+ is harmonic the coefficients are connected via the relations

$$\begin{aligned}B_1 + B_2 + B_3 &= 0 \qquad (14)\\ 3\Gamma_1 + \Gamma_{21} + \Gamma_{31} &= 0 \qquad (15)\\ \Gamma_{12} + 3\Gamma_2 + \Gamma_{32} &= 0 \qquad (16)\\ \Gamma_{13} + \Gamma_{23} + 3\Gamma_3 &= 0 \qquad (17)\end{aligned}$$

Hence, among the 19 coefficients appearing in the expressions (13) only 15 are linearly independent, and since the seeking numbers are only 13 it follows that, at least in principle, the information carried by the terms $n = 1, 2, 3$ of the expansion (6) is enough to identify the source in its localized form (8). So, what we actually need to do, in order to solve the inverse problem and identify the source, is to utilize the algebraic relations which associate the coefficients in (9) with the corresponding known expressions that these coefficients have to have from the relations (10)-(12) and to solve the resulting nonlinear algebraic system. Nevertheless, this program leads to a system which either is extremely difficult, or it is impossible, to be solved analytically. However, we can consider a special, but not trivial case, which does preserve the essential stracture of the problem and at the same time it is analytically tractable. In fact, we assume that the point $\boldsymbol{r}_0$ is located on the x_3–axis, and that the line segment is oriented along the x_1–axis. That is

$$\boldsymbol{r}_0 \quad = \quad (0,0,r_0)\text{ and }\hat{\boldsymbol{a}} = (1,0,0). \qquad (18)$$

This system is obtained through a series of analytic calculations and reads as follows:

$$(A_1, A_2, A_3) = 6L(Q_1, Q_2, Q_3) \quad (19)$$

$$B_1 = -\frac{5}{6}A_3 r_0 + \frac{10}{3}L^3 l_1 \quad (20)$$

$$B_2 = -\frac{5}{6}A_3 r_0 - \frac{5}{3}L^3 l_1 \quad (21)$$

$$B_3 = \frac{5}{3}A_3 r_0 - \frac{5}{3}L^3 l_1 \quad (22)$$

$$B_{12} = 5L^3 l_2 \quad (23)$$

$$B_{23} = \frac{5}{2}A_2 r_0 \quad (24)$$

$$B_{31} = \frac{5}{2}A_1 r_0 + 5L^3 l_3 \quad (25)$$

$$\Gamma_1 = \frac{7}{9}L^2 A_1 - \frac{14}{3}L^3 l_3 r_0 - \frac{7}{6}A_1 r_0^2 \quad (26)$$

$$\Gamma_2 = -\frac{7}{18}L^2 A_2 - \frac{7}{6}A_2 r_0^2 \quad (27)$$

$$\Gamma_3 = -\frac{7}{18}L^2 A_3 - \frac{14}{3}L^3 l_1 r_0 + \frac{7}{3}A_3 r_0^2 \quad (28)$$

$$\Gamma_{12} = \frac{14}{9}L^2 A_2 - \frac{7}{6}A_2 r_0^2 \quad (29)$$

$$\Gamma_{21} = -\frac{7}{6}L^2 A_1 - \frac{14}{3}L^3 l_3 r_0 - \frac{7}{6}A_1 r_0^2 \quad (30)$$

$$\Gamma_{13} = \frac{14}{9}L^2 A_3 + \frac{56}{3}L^3 l_1 r_0 - \frac{7}{2}A_3 r_0^2 \quad (31)$$

$$\Gamma_{31} = -\frac{7}{6}L^2 A_1 + \frac{56}{3}L^3 l_3 r_0 + \frac{14}{3}A_1 r_0^2 \quad (32)$$

$$\Gamma_{23} = -\frac{7}{18}L^2 A_3 - \frac{14}{3}L^3 l_1 r_0 - \frac{7}{2}A_3 r_0^2 \quad (33)$$

$$\Gamma_{32} = -\frac{7}{18}L^2 A_2 + \frac{14}{3}A_2 r_0^2 \quad (34)$$

$$\Gamma_{123} = \frac{70}{3}L^3 l_2 r_0. \quad (35)$$

IV. The inverse problem

All we have to do now is to elliminate the parameters l_1, l_2, l_3 between the equations (19)-(35) and then to solve the reduced system with respect to the seeking quandities $Q_1, Q_2, Q_3, r_0,$ and L. Since we have 19 equations connecting 8 unknowns, the determination of the unknowns can be done in many different ways. In any case, the final result is given by

$$r_0 = \frac{2}{5}\frac{B_{23}}{A_2} \quad (36)$$

$$L = \left(\frac{9}{14}\frac{\Gamma_{12}}{A_2} + \frac{3}{25}\left(\frac{B_{23}}{A_2}\right)^2\right)^{1/2} \quad (37)$$

$$Q_i = \frac{A_i}{6}\left(\frac{9}{14}\frac{\Gamma_{12}}{A_2} + \frac{3}{25}\left(\frac{B_{23}}{A_2}\right)^2\right)^{-1/2}, \quad i = 1, 2, 3. \quad (38)$$

So, the importance of this example is that it demonstrates that the support of a localized neuronal current is in fact identifiable, as long as its dimension is less than three.

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