Analysis of relative errors and bounds in localization of dipoles in various ellipsoidal brain models in Encephalography

George Dassios, Michael Doschoris, Fotini Kariotou and Vasiliki-Christina Panagiotopoulou.

***Abstract*— Electroencephalography (EEG) measures potential differences on part of the surface of the head. These measurements are directly connected with activated regions within the brain, modeled as dipoles, and are accurately interpreted if originating from a average ellipsoidal conductor with semi-axes 5.5, 6.5 and 8.5 $\times 10^{-2}$ m. However, the volume of modern human brains varies significantly depending on sex and age. These variations in volume could introduce a source of error affecting the location of the dipole if not incorporated in existing models. In what follows, an error estimation is established for EEG readings in the case where the average ellipsoidal brain is replaced by an ellipsoid with different volume.**

I. INTRODUCTION

The imaging techniques which allow the study of the electromagnetic brain activity as a real time process are electroencephalography (EEG) and magnetoencephalography (MEG). Due to their non-invasiveness, both techniques are well established and of great medical significance, but rely on accurate algorithms which must efficiently handle the corresponding inverse problems. As a rule, analytic inversion algorithms are developed approximating the Head-brain system as a spherical conductor. The necessary parameter(s) related to the geometry of the conductor are determined on the basis of MRI-volumetric studies (see for example [1]-[5]), almost always providing only statistical mean values.

The volume of modern human brains, however, considerable varies among individuals, ranging between 1.053 $\times 10^{-3}$ m^3 and 1.499 $\times 10^{-3}$ m^3 for males (average: 1.274 $\times 10^{-3}$ $\pm$ 0.115$\times 10^{-3}$ m^3) whereas for females the corresponding values are 0.975 $\times 10^{-3}$ m^3 and 1.398 $\times 10^{-3}$ m^3 (average: 1.131 $\times 10^{-3}$ $\pm$0.099 $\times 10^{-3}$ m^3) [1]. Moreover, because the main effort of scientist and engineers is focused in developing accurate analytic algorithms the use of the ellipsoidal geometry is unsurpassed.

In details, as literature review reveals errors and bounds for EEG data are subjects of great interest for numerous studies [6]-[10]. Most of publications use other numerical approximations [7], [9] or spherical geometry for the shpae of the head. As for the error method used, this could range from simple mean square calculations [9] in localizing the dipole position to complicated Bayesian methods [8]. In these cases, errors could range from 2 $\times 10^{-3}$ m to 25 $\times 10^{-3}$ m [6]-[10] depending on the method. These findings underline the need for accuracy in EEG data and provide space for research towards the decrease of errors from EEG measurements.

An important question regarding the modalities of Electro- and Magnetoencephalography is to establish how sensitive standard inversion algorithms are in the case where measurements are wrongfully interpreted by arriving from an average ellipsoidal brain. The present analysis consists the first step towards this direction.

The article is organized as follows. An elementary introduction to the theory of ellipsoidal harmonics as well as to the forward EEG problem are provided in the sequel. These notions are then implemented to compute the errors and bounds associated with measurements originating from a ellipsoidal volume conductor different from the so-called reference ellipsoid.

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II. MATHEMATICAL FORMULATION

A. The Ellipsoidal Coordinate System.

In the ellipsoidal coordinate system (ρ, μ, ν) each point is specified by the intersection of three non-degenerate second-degree surfaces, corresponding to an ellipsoid, a hyperboloid of one sheet as well as a hyperboloid of two sheets. The family of confocal ellipsoids are represented via

$$\frac{x_1^2}{\rho^2} + \frac{x_2^2}{\rho^2 - h_3^2} + \frac{x_3^2}{\rho^2 - h_2^2} = 1, \; \rho^2 \in (h_2^2, +\infty) \quad (1)$$

whereas the remaining two surfaces are described by equations identical to (1) interchanging ρ by the variables μ and ν, respectively, defined on the intervals $\mu^2 \in (h_3^2, h_2^2)$ and $\nu^2 \in (0, h_3^2)$. The constants

$$h_1^2 = a_2^2 - a_3^2, \quad h_2^2 = a_1^2 - a_3^2, \quad h_3^2 = a_1^2 - a_2^2$$

are the squares of the semi-focal distances and a_i, $i = 1, 2, 3$ with $0 < a_3 < a_2 < a_1 < +\infty$ fixed parameters determining the reference semi-axes.

Further, the ellipsoidal coordinates of every point are connected to the Cartesian by the following relations

$$x_1 = \frac{\rho\,\mu\,\nu}{h_2\,h_3}, \quad (2)$$

$$x_2 = \frac{\sqrt{\rho^2 - h_3^2}\,\sqrt{\mu^2 - h_3^2}\,\sqrt{h_3^2 - \nu^2}}{h_1\,h_3}, \quad (3)$$

$$x_3 = \frac{\sqrt{\rho^2 - h_2^2}\,\sqrt{h_2^2 - \mu^2}\,\sqrt{h_2^2 - \nu^2}}{h_1\,h_2}, \quad (4)$$

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provided that $0 \leq \nu^2 \leq h_3^2 \leq \mu^2 \leq h_2^2 \leq \rho^2$.

Equation (1) as well as the two equations arising interchanging ρ by the variables μ and ν, respectively, demonstrate the unique character of every direction in the ellipsoidal system and in order to establish the variations in angular dependence, a reference ellipsoid has to be introduced, a direct analogy to the unit sphere.

The reference ellipsoid $\mathcal{E}_{\text{ref}}$ is defined via (1) replacing ρ by a_1, i.e.

$$\mathcal{E}_{\text{ref}}: \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} = 1. \tag{5}$$

We emphasize, that in order to solve boundary value problems in the ellipsoidal coordinate system it is essential to adopt an ellipsoid in such a way as to fit the actual boundary by choosing a particular value of ρ. This is secured if we use the boundary of our domain to be the reference ellipsoid (5) and construct the ellipsoidal system that is based on it.

In closing this subsection, we note that the ellipsoidal coordinate system, by its very nature, is demanding, concealing numerous difficulties. Therefore, the authors strive to provide all necessary details regarding the development. However, due to the limited space this is beyond the bounds of possibility and as a result the interested reader is referred to [4].

B. *The Forward EEG Problem.*

Activation of a localized region in the brain triggers a primary neuronal current $\mathbf{J}^{\text{p}}(\mathbf{r})$ leading to a measurable electric potential on the boundary.

Consider a homogeneous ellipsoidal conductor with semi-axes a_i, $i = 1, 2, 3$ and conductivity σ, which takes the place of the reference ellipsoid for our ellipsoidal coordinate system.

In the case where the neuronal current is represented by a single equivalent dipole at the point $\mathbf{r}_0 = (\rho_0, \mu_0, \nu_0)$ with moment $\mathbf{Q}$, then $\mathbf{J}^{\text{p}}(\mathbf{r}) = \mathbf{Q}\,\delta(\mathbf{r} - \mathbf{r}_0)$, δ denoting the Dirac measure.

The *interior* electric potential $u^-(\rho, \mu, \nu)$ is obtained by solving the following Neumann problem

$$\begin{aligned} &\Delta\, u^-(\rho, \mu, \nu) = \frac{1}{\sigma}\mathbf{Q} \cdot \nabla\delta(\mathbf{r} - \mathbf{r}_0), \quad h_2 \leq \rho < a_1, \\ &\frac{\partial}{\partial\rho}\, u^-(\rho, \mu, \nu) = 0, \quad \rho = a_1, \end{aligned}$$

whereas the corresponding electric potential $u^+(\rho, \mu, \nu)$ *outside* the head is derived by the Dirichlet problem

$$\begin{aligned} &\Delta\, u^+(\rho, \mu, \nu) = 0, \quad \rho > a_1, \\ &u^+(\rho, \mu, \nu) = u^-(\rho, \mu, \nu), \quad \rho = a_1, \\ &u^+(\rho, \mu, \nu) = \mathcal{O}\left(\frac{1}{r^2}\right), \quad \mathbf{r} \to \infty. \end{aligned}$$

After tedious but straightforward calculations (details regarding the derivation of the formulas can be found in [3]), we find

$$u^{\pm}(\mathbf{r}) = \frac{1}{\sigma} \lim_{N\to\infty} \sum_{n=1}^{N} u_n^{\pm}(\mathbf{r}), \tag{6}$$

where the partial sums $u_n^{\pm}(\mathbf{r})$ are defined as

$$\begin{aligned} u_n^-(\rho, \mu, \nu) = \sum_{m=1}^{2n+1} \frac{\mathbf{Q} \cdot \nabla_{\mathbf{r}_0}\, \mathbb{E}_n^m(\mathbf{r}_0)}{\gamma_n^m} \Bigg(I_n^m(\rho) - I_n^m(a_1) \\ + \frac{1}{a_2\, a_3\, E_n^m(a_1)\, \frac{\mathrm{d}}{\mathrm{d}\rho} E_n^m(a_1)} \Bigg)\, \mathbb{E}_n^m(\rho, \mu, \nu), \end{aligned}$$

and

$$u_n^+(\rho, \mu, \nu) = \frac{1}{a_2\, a_3} \sum_{m=1}^{2n+1} \frac{\mathbf{Q} \cdot \nabla_{\mathbf{r}_0}\, \mathbb{E}_n^m(\mathbf{r}_0)}{\gamma_n^m\, F_n^m(a_1)\, \frac{\mathrm{d}}{\mathrm{d}\rho} E_n^m(a_1)}\, \mathbb{F}_n^m(\rho, \mu, \nu),$$

respectively. On the other hand, on the boundary of the reference ellipsoid $\mathcal{E}_{\text{ref}}$ we have

$$u_n(a_1, \mu, \nu) = \frac{1}{a_2\, a_3} \sum_{m=1}^{2n+1} \frac{\mathbf{Q} \cdot \nabla_{\mathbf{r}_0}\, \mathbb{E}_n^m(\mathbf{r}_0)}{\gamma_n^m\, \left(\frac{\mathrm{d}}{\mathrm{d}\rho} E_n^m(a_1)\right)}\, S_n^m(\mu, \nu). \tag{7}$$

Here, E_n^m denote the Lamé functions of the first kind, divided into four different classes, known as K, L, M and N. The structure of each class is as following

$$\begin{aligned} &K(x) = x^n + c_1\, x^{n-2} + c_2\, x^{n-4} + \ldots, \\ &L(x) = \sqrt{|x^2 - h_3^2|}\, \left(x^{n-1} + d_1\, x^{n-3} + \ldots\right), \\ &M(x) = \sqrt{|x^2 - h_2^2|}\, \left(x^{n-1} + f_1\, x^{n-3} + \ldots\right), \\ &N(x) = \sqrt{|x^2 - h_3^2|}\, \sqrt{|x^2 - h_2^2|}\, \left(x^{n-2} + g_1\, x^{n-4} + \ldots\right), \end{aligned}$$

x standing for either variable ρ, μ, ν. The corresponding Lamé functions of the second kind $F_n^m(\rho)$ are

$$\begin{aligned} &F_n^m(\rho) = (2n + 1)\, E_n^m(\rho)\, I_n^m(\rho), \\ &I_n^m(\rho) = \int_\rho^\infty \frac{\mathrm{d}t}{\left(E_n^m(t)\right)^2 \sqrt{t^2 - h_3^2}\sqrt{t^2 - h_2^2}}, \quad \rho \geq h_2 \end{aligned}$$

where I_n^m is the elliptic integral.

On the other hand, the product of two Lamé functions belonging to the same class define the surface ellipsoidal harmonics S_n^m, i.e.

$$S_n^m(\mu, \nu) = E_n^m(\mu) E_n^m(\nu),$$

whereas

$$\mathbb{E}_n^m(\rho, \mu, \nu) = E_n^m(\rho) S_n^m(\mu, \nu) \tag{8}$$

designate the interior ellipsoidal harmonics (ellipsoidal harmonics of the first kind). Similar, the exterior ellipsoidal harmonics are

$$\mathbb{F}_n^m(\rho, \mu, \nu) = (2n + 1)\, \mathbb{E}_n^m(\rho, \mu, \nu)\, I_n^m(\rho).$$

Last but not least, γ_n^m indicate the normalization constants, being proportional to the number π, given by the formula

$$\gamma_n^m = \oint_{\mathrm{S}_{a_1}} \left(S_n^m(\mu, \nu)\right)^2 \mathrm{d}\Omega(\mu, \nu)$$

for every $n = 0, 1, 2, \ldots$ and $m = 1, 2, \ldots, 2n + 1$ where the ellipsoidal solid angle element $\mathrm{d}\Omega(\mu, \nu)$ is independent of the ellipsoidal surface specified by the variable ρ.

C. Approximation Error and Bounds

Let us introduce a second ellipsoid $\mathcal{E}_2$ which is completely determined by its semi-axes $\bar{a}_i$, $i = 1,2,3$ defined by the equation

$$\mathcal{E}_2 : \frac{x_1^2}{\bar{a}_1^2} + \frac{x_2^2}{\bar{a}_2^2} + \frac{x_3^2}{\bar{a}_3^2} = 1. \tag{9}$$

Further, allow $\mathcal{E}_2$ to be confocal to the reference ellipsoid $\mathcal{E}_{\text{ref}}$ defined by (5), therefore differing only by a parameter ϵ such that $\rho = a_1 + \epsilon$.

With that understanding, equation (9) becomes

$$\mathcal{E}_2 : \frac{x_1^2}{(a_1+\epsilon)^2} + \frac{x_2^2}{(a_1+\epsilon)^2 - h_3^2} + \frac{x_3^2}{(a_1+\epsilon)^2 - h_2^2} = 1. \tag{10}$$

Comparing relations (5) and (10) implies the following formulae connecting the semi-axes a_i and $\bar{a}_i$, $i = 1,2,3$

$$\bar{a}_1 = a_1 + \epsilon, \tag{11}$$

$$\bar{a}_2 = \sqrt{(a_1+\epsilon)^2 - h_3^2}, \tag{12}$$

$$\bar{a}_3 = \sqrt{(a_1+\epsilon)^2 - h_2^2}. \tag{13}$$

The notion of confocal ellipsoids permits application of relation (7) on the boundary of $\mathcal{E}_2$ as well by simply substituting a_i, $i = 1,2,3$ by their over lined counterparts. Subtracting in the sequel these electric potentials and taken into consideration only first degree terms, gives

$$u_1(a_1,\mu,\nu) - u_1(\bar{a}_1,\mu,\nu) = \frac{1}{\sigma}\sum_{m=1}^{3} \frac{\mathbf{Q}\cdot\nabla_{\mathbf{r}_0}\mathbb{E}_1^m(\mathbf{r}_0)}{\gamma_1^m} \times S_1^m(\mu,\nu)\left[\frac{1}{a_2\,a_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(a_1)\right)} - \frac{1}{\bar{a}_2\,\bar{a}_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(\bar{a}_1)\right)}\right]. \tag{14}$$

Note that,

$$\frac{1}{a_2\,a_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(a_1)\right)} > \frac{1}{\bar{a}_2\,\bar{a}_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(\bar{a}_1)\right)}$$

if ϵ is positive or, if $\epsilon < 0$

$$\frac{1}{a_2\,a_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(a_1)\right)} < \frac{1}{\bar{a}_2\,\bar{a}_3\left(\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(\bar{a}_1)\right)}.$$

It is of interest to write above expression in terms of Cartesian coordinates. To this end, we first calculate the source dependent directional derivative $\mathbf{Q}\cdot\nabla_{\mathbf{r}_0}$ on the eigenfunctions $\mathbb{E}_1^m(\mathbf{r}_0)$. The Lamé functions for $n = 1$ are

$$E_1^m(x) = \sqrt{|x^2 - (a_1^2 - a_m^2)|}, \quad m = 1,2,3 \tag{15}$$

where again x stands for either variable ρ,μ,ν, and the following relation is easily verified

$$\frac{\mathbf{Q}\cdot\nabla_{\mathbf{r}_0}\mathbb{E}_1^m(\mathbf{r}_0)}{\gamma_1^m} = \frac{3}{4\pi}\frac{1}{h_1\,h_2\,h_3}h_m\,Q_m, \quad m = 1,2,3 \tag{16}$$

provided that

$$\gamma_1^m = \frac{4\pi}{3}\left(\frac{h_1\,h_2\,h_3}{h_m}\right)^2.$$

We note that relation (16) is independent of the dipoles location.

Furthermore,

$$E_1^m(a_1) = a_m, \quad m = 1,2,3,$$

$$\frac{\mathrm{d}}{\mathrm{d}\rho}E_1^m(a_1) = \frac{a_1}{a_m}, \quad m = 1,2,3,$$

and

$$S_1^m(\mu,\nu) = \frac{h_1\,h_2\,h_3}{a_m\,h_m}x_m, \quad m = 1,2,3.$$

Next, one has to combine (15) with (11)-(13) and expand the resulting expressions as Taylor series regarding the parameter ϵ. Putting everything back into (14) gives

$$u_1(a_1,\mu,\nu) - u_1(\bar{a}_1,\mu,\nu) = \epsilon\,\frac{a_1}{\sigma\,V_{\text{ref}}} \times \sum_{m=1}^{3} Q_m\,x_m\left(\frac{1}{a_1^2} + \frac{1}{a_2^2} + \frac{1}{a_3^2} - \frac{1}{a_m^2}\right), \tag{17}$$

where

$$V_{\text{ref}} = \frac{4\pi}{3}a_1\,a_3\,a_3$$

is the volume of the reference ellipsoid.

The relative error of the latter is easily computed bearing in mind that

$$u_1(a_1,\mu,\nu) = \frac{1}{\sigma\,V_{\text{ref}}}\sum_{m=1}^{3} x_m\,Q_m.$$

Moreover, without loss of generality we consider $Q_m = Q$ whereas for the reference ellipsoid $\mathcal{E}_{\text{ref}}$ we have $|x_m| \leq a_m$, $m = 1,2,3$, leading to the following local bound

$$\frac{|u_1(a_1,\mu,\nu) - u_1(\bar{a}_1,\mu,\nu)|}{|u_1(a_1,\mu,\nu)|} \leq \frac{|\epsilon|}{||x_1| - |x_2| - |x_3||} \times a_1\sum_{m=1}^{3} a_m\left(\frac{1}{a_1^2} + \frac{1}{a_2^2} + \frac{1}{a_3^2} - \frac{1}{a_m^2}\right) \tag{18}$$

subject to

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} = 1. \tag{19}$$

III. Numerical Examples

The average, minimum and maximum volume of the human brain is $V_{\text{aver}} = 1.272 \times 10^{-3}\,\text{m}^3$, $V_{\text{mim}} = 1.053 \times 10^{-3}\,\text{m}^3$ and $V_{\text{max}} = 1.5 \times 10^{-3}\,\text{m}^3$, respectively, corresponding to semi-axes $(a_1,a_2,a_3) = (8.5, 6.5, 5.5)$ $\times 10^{-2}$ m, $(\bar{a}_{1,\text{min}},\bar{a}_{2,\text{min}},\bar{a}_{3,\text{min}}) = (7.5, 6.7, 5)$ $\times 10^{-2}$ m and $(\bar{a}_{1,\text{max}},\bar{a}_{2,\text{max}},\bar{a}_{3,\text{max}}) = (9, 6.5, 6)$ $\times 10^{-2}$ m.

Remark 3.1: Although, by implementing aforementioned values relation (11) supplies an interval in which the parameter ϵ lives, namely $\epsilon \in [-1, 0.5]$ $\times 10^{-2}$ m, great care must be taken by using them. We recall that in deriving relation (17)

TABLE I

MAXIMUM PERCENTAGE OF RELATIVE LOCAL ERROR. THE PARAMETER ϵ IS SET TO $0.1\ \times 10^{-2}$ M

x_1 $\times 10^{-2}$ m	x_2 $\times 10^{-2}$ m	x_3 $\times 10^{-2}$ m	error %
-8.5	-5.0	4.6	42
-8.5	-3.5	30	34
-8.5	-1.5	1.3	12
-2.5	6.5	1.6	12
-3.5	6.5	2.3	13
-5.5	6.5	3.6	15
0.0	-1.5	5.3	10
0.0	-2.5	5	9
0.0	-3.5	4.6	9
6.0	0.0	3.9	33
7.0	0.0	3.1	18

we extensively used Taylors expansion. Obviously, (17) and formulas based on it are only valid in the common region of convergence, computed with the aid of the corresponding nth term of each series. There exactly lies the difficulty. These terms are almost impossible to obtain. However, relation (17) remains accurate in a small region around zero.

The relative error bounds computed via (18) for various points on the surface of the reference ellipsoid for fixed parameter $\epsilon = 0.1\ \times 10^{-2}$ m are shown in Table I.

IV. CONCLUSIONS AND FUTURE WORKS

Despite the noticeable diversity of the size of human brains, we insist on treating them as average when it comes to measure and analyse them. In other words, inversion algorithms base their calculations on parameters tailored to an equivalent sphere or rarely to an ellipsoid.

A first step towards the answer has been provided in the framework of the present article. Using only first degree terms the maximum error, strongly depending on the measurement site, has been evaluated (see Table I). However, the indicated terms are inadequate for two major reasons: (a) The action of the source dependent operator $\mathbf{Q} \cdot \nabla_{\mathbf{r}_0}$ on the eigenfunctions $\mathbb{E}_n^m(\mathbf{r}_0)$ (16) does not incorporate the location of the source. (b) The expression of the matching magnetic field involves no terms of the first degree. The latter implies that the magnetic potential measured a few centimeters away from the surface of the head is also zero.

Obviously, future work must include higher degree terms. Although this sounds trivial if working in the spherical coordinate system, in the ellipsoidal geometry this is a deficiency associated with the lack of general formulas for Lamé functions and related Ellipsoidal harmonics [11].

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