3D Printing: Basic concepts Mathematics and Technologies

Athanasios Anastasiou, Charalambos Tsirmpas, Alexandros Rompas, Kostas Giokas, Dimitris Koutsouris

School of Electrical and Computer Engineering National Technical University of Athens, Biomedical Engineering Laboratory, Athens, Greece

***Abstract*: 3D printing is the process of being able to print any object layer by layer. But if we question this proposition, can we find any three dimensional objects that can't be printed layer by layer? To banish any disbeliefs we walked together through the mathematics that prove 3d printing is feasible for any real life object. 3d printers create three dimensional objects by building them up layer by layer. The current generation of 3d printers typically requires input from a CAD program in the form of an STL file, which defines a shape by a list of triangle vertices. The vast majority of 3d printers use two techniques, FDM (Fused Deposition Modelling) and PBP (Powder Binder Printing). One advanced form of 3d printing that has been an area of increasing scientific interest the recent years is bioprinting. Cell printers utilizing techniques similar to FDM were developed for bioprinting. These printers give us the ability to place cells in positions that mimic their respective positions in organs. Finally through series of case studies we show that 3d printers in medicine have made a massive breakthrough lately.**

Keywords: 3D Printing, Fubini Theorem, FDM,PBP, Bioprinting, Organ Printing, Human heart, Bionic ear, Skull lesions.

I. Introduction

3d printing is a rapidly developing technology in the last years. This industrial revolution has applications in the fields of engineering, medicine and many more. These include creation of mass-customized products, prototypes, replacement parts and even medical and dental implants. The speed and ease of designing and modifying products has made them the number one rapid prototyping technique. The aim of this article is to evaluate this matter from another point of view by focusing on their contribution to the fields of medicine. We will first examine the mathematical knowledge behind 3d printing through the opinion of experts. Next we will give a short description of the most famous 3d printing methods highlighting one special form, bioprinting. Finally we will focus on a series of case studies where 3d printers where used to guide surgeons, create enhanced human parts and mend human skull defects.

Manuscript received July 26, 2013. This work was supported by Biomedical Engineering Laboratory of the National Technical University of Athens.

Athanasios Anastasiou is with the National Technical University of Athens, in the Biomedical Engineering Laboratory, Athens, Greece (e-mail:aanastasiou@biomed.ntua.gr).

Charalambos Tsirmpas is with the National Technical University of Athens, in the Biomedical Engineering Laboratory, Athens, Greece (e-mail:htsirbas@biomed.ntua.gr).

Alexandros Rompas is with the National Technical University of Athens, in the Biomedical Engineering Laboratory, Athens, Greece (corresponding author, e-mail:arompas@biomed.ntua.gr).

Kostas Giokas is with the National Technical University of Athens, in the Biomedical Engineering Laboratory, Athens, Greece (e-mail:kgiokas@biomed.ntua.gr).

Dimitris Koutsouris is with the National Technical University of Athens, in the Biomedical Engineering Laboratory, Athens, Greece (e-mail:dkoutsou@biomed.ntua.gr).

II. 3D Printing Made Real: Fubini Theorem

Before we turn our focus on printing a three-dimensional model we have to consider what mathematical statements and theorems allow us to materialize this idea. Practically 3d printing is about being able to print any object layer by layer. But if we question this belief, can we find any three-dimensional objects that can't be printed layer by layer?

So next we will analyse the theorem that proves 3d printers can duplicate everything (any real life physical object at least). Fubini's theorem, named after the Italian mathematician Guido Fubini, states that an object of n dimensions can be represented as a spectrum of layers of shapes of n-1 dimensional layers. This means that a 3 dimensional shape (any shape in the real world) can be portrayed as layers of 2 dimensional shapes [1]. In 3d printing technology this means that we are able to express any 3d object as layers of 2d planes. Below we provide the theorem but not its proof since doesn't serve the purpose of this article. [2]

In order to analyse the theorem, we Suppose A and B are complete measure spaces. Supposes f(x,y) is A x B measurable. If

$$\int_{AxB} |f(x,y)|\, d(x,y) < \infty \quad (1)$$

Where the integrals is taken with respect to a product measure on the space over A x B, then

$$\int_A \left(\int_B f(x,y)\, dy\right) dx = \int_B \left(\int_A f(x,y)\, dx\right) dy = \int_{AxB} f(x,y)\, d(x,y) \quad (2)$$

The first two integrals being iterated integrals with respect to two measures, respectively and the third being an integral with respect to a product of these two measures.

If the above integral of the absolute value is not finite, then the two iterated integrals may actually have different values. Thus we have that:

Fi f(x,y)=g(x)h(y) for some functions g and h, then

$$\int_A g(x)dx \int_B h(y)dy = \int_{AxB} f(x,y)\, d(x,y) \quad (3)$$

The integral on the right side being with respect to a product measure.

Fubini's theorem proves that 3d printers can print any real life objects. However, a practical limitation is the slicing resolution and also the achievement of physical stability during layering [1].

III. Fused-Deposition-Modelling

FDM 3d printers operate by building layers via the extrusion of thin semi molten plastic beads, usually ABS (Acrylonitrile Butadiene Styrene) plastic. This material is quite attractive for its properties since it has low toxicity levels, is highly durable and hard. It can be dyed with different colours

978-1-4799-3163-7/13/$31.00 ©2013 IEEE

nevertheless ABS is typically used in its natural off-white form. One of its disadvantages is that it comes out soft thus any overhanging parts need to be supported with the appropriate structures until it hardens [2]. FDM printers are known for their ability to print strong and precise objects that can be used for many applications. The process described, is shown in Figure 1 below.

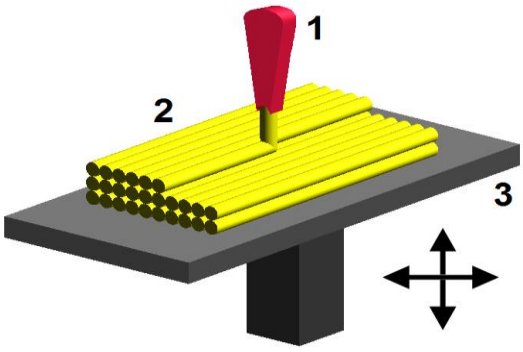

Figure 1. Fused deposition modeling: 1 - nozzle ejecting molten plastic, 2 - deposited material (modeled part), 3 - controlled movable table

IV. POWDER -BINDER PRINTING

This technique works by building up layers of a plaster-like powder that is sprayed by a liquid binder, or glue from an ink-jet printer head. In every pass a new layer of loose powder followed by the binder spray is applied [2]. In order to understand these processes we can visualize each layer of powder as a piece of paper. Having said that this technique doesn't differ significantly from conventional 2d ink-jet printing except that each layer fuses with the previous ones. Any remaining powder not sprayed with binder is removed to be recycled for further use after the end of the process. One of the greater advantages of this method is that the powder can hold all every overhanging part of the structure in place, so no supports are needed. However, it may be required that some parts of the printed object still need support in case they are long overhanging or too thin [7].

V. BIOPRINTING

An alternative type of 3d printing with increasing academic interest is bioprinting. Bioprinting, as described in the International Conference of Bioprinting and Biofabrication in Bordeaux, is ”the use of computer-aided transfer processes for patterning and assembling living and non-living materials with a prescribed 2D or 3D organization in order to produce bio-engineered structures serving in regenerative medicine, pharmacokinetic and basic cell biology studies” [4]. For bioprinting purposes, cell printers utilizing similar techniques to FDM were developed. These printers give us the ability to place cells in areas that mimic their respective coordinates in organs. The capability to drop cells on previously printed successive layers provides an opportunity for three dimensional organ printing [3]. Such breakthrough of bioprinting technology in three dimensions draws from the use of thermo-reversible gels. Gels are fluid at 20ºC and above 32ºC and therefore similar to conventional printing methods can be applied, so as tissue structures can be printed with cells representing biological ink. According to the above, successive layers could be produced just by dropping another layer of gel onto an already printed surface.

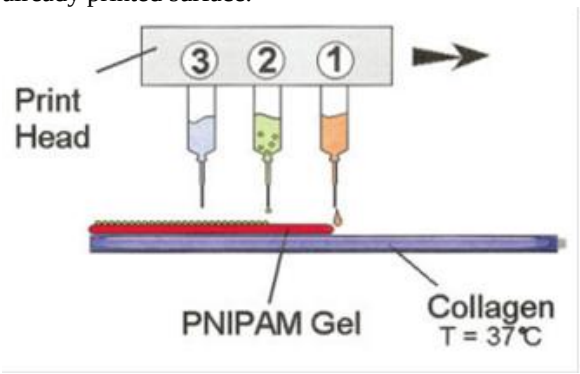

Figure 2. Principle of organ printing [3]

This technology allows us to print 3D complex organs with accurate and precise assignment of various cell types to the gel. Feasibility of this technology can be shown, if we take into consideration that human cells are placed close enough in sequential layers of 3d gels, which can be fused and create fully functional organs and cultured in vitro. The process is illustrated in Figures 3 and 4 below.

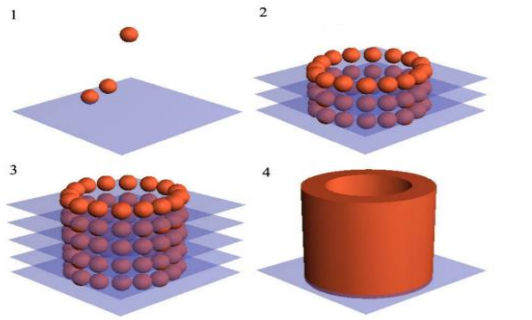

Figure 3.The Mathematical model of cell aggregate behavior when implanted in a 3D model gel [8].

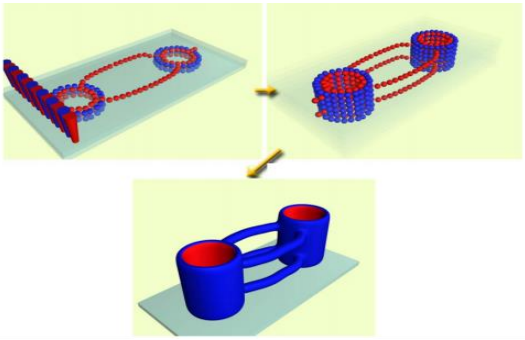

Figure 4. Tissue self-assembly after the careful placement of cells in the original geometrical positions [8].

This principle of cell self-assembly into fully vascularized tissues is similar to the way embryonic like-issues sort and fuse into functional forms dictated by the rules of developmental biology. Another approach to the creation of living tissues and organs through bioprinting is based on mathematical modelling using a set of theoretical principles, rules or laws related to spatial organization [11]. While this idea appears promising, some issues were raised by the Global Medical Society with regard to cell survival, tissue perfusion and vascularization. Material issues are of utmost importance as well since they can enhance the whole process but also negatively influence cell fate [3], [8].

VI. CASE STUDY AND SCENARIO

Taking into account an efficient surgical plan, information is extracted via CT and MR images. These images provide doctors with sufficient information regarding the patient's anatomical structure. Followed by the analysis of two-dimensional images, surgeons can generate a more careful intraoperative procedure. The understanding internal and external structure of organs could potentially be magnified, if doctors have in hand a real three-dimensional object illustration instead of a virtual one. Innovative technologies such as prototyping can produce different methods of organ replication based on patient-specific data. Therefore, if the procedure is meticulous and successful the result precisely illustrates the defection of the organ under examination. Below, we describe the steps followed for the creation of a human heart showing a congenital defect.

Prior to implementation, a visual three-dimensional representation of heart must be plotted in a computer. Thus, the data extracted from CT or MRI images are processed in order to obtain a three-dimensional model depicting the desired structures. All the above require a deep understanding in the field of digital image processing (segmentation, region growing, smoothing) resulting a VRML (Virtual Reality Markup Language) file which is imported in 3D printer as input so the desired object can be produced.

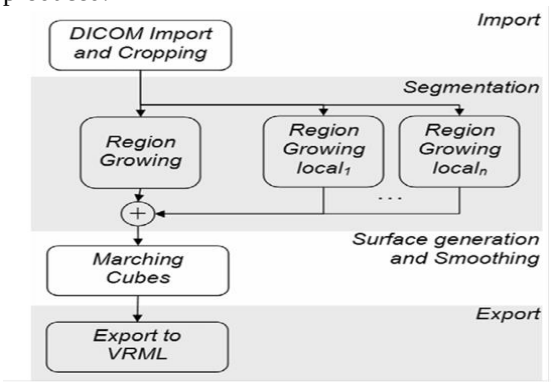

Figure 5. Workflow network of processing the image data to obtain a 3D model suitable for the RP process. The result of the segmentation is improved by applying local segmentation processes with adjusted threshold values. After the surface has been generated, the data are exported to VRML data format [7].

More precisely, the VRML file containing the virtual representation of the three dimensional model is loaded into the controlling system of the 3D printer. The 3D printer produces the model from starch by slices, each having a height of 0.2mm. Each slice is rolled out to the building area fetching the powder from the feed tray. Once, the resulting model is in position the powder is fixed by a binder with the use of inkjet technology. In the same fashion as common printers, all colours can be produced in inkjet printers by mixing cyan, magenta and yellow binder.

Vascular structures have a great risk of breaking or falling apart during the manufacturing process if the stability of the object is not guaranteed. During the printing the model is surrounded with loose powder. The amount of binder sprayed onto the powder is reduced within the interior of the model, so that the model's weight is decreased but its surface is kept solid. Following this process our structure is protected from having its sensitive parts cracked, broken or deformed. When the last layer is applied and construction is completed, our 3d prototype is left to dry for approximately 60 minutes. Subsequently, any loose powder surrounding our model is removed with the use of an air jet. Before proceeding any further our printed model is left to dry for 4-6 hours depending on its size at 70o C.

In order to achieve the required stability while avoiding having a stiff 3D model we filter an elastomer based on polyurethane allowing for high flexibility characteristics. Additional layers of elastomer will follow, in order to stabilize and smoothen the model without reducing its flexibility. Finally, the parts of prototype are bent. The remaining starch in the interior breaks and can be removed giving us the final printed-model consisting only from elastomer. To remove the remaining starch we have to flush it out. For that reason, a hole is drilled into the surface of the 3D model with a diameter of at least 1mm. Next the printed model is left in a vessel full of water for 6 hours allowing the starch to be resolved into the water and flushed out. The final result is showed in the image below [7].

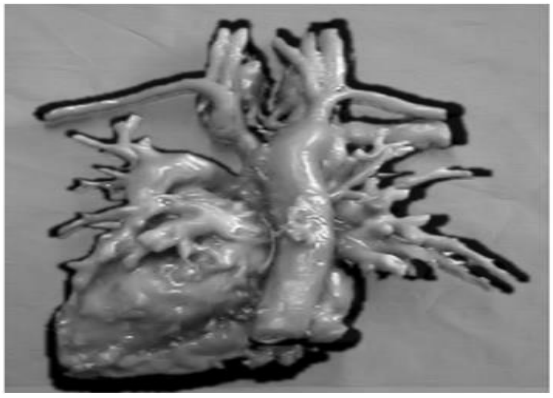

Figure 6. A beating heart model [7].

It is worth mentioning that the resulting printed-model described above is patient-specific. Therefore, if we account for a new patient, the whole procedure must be followed

from scratch in order to include the unique anatomical characteristics.

Compared to a two-dimensional printed object, in 3D representations dimensions and distances among structures can be conveniently examined. Moreover, the 3D printing technique used offers resolution less than 0.01 mm in horizontal directions and about 0.2 mm in vertical directions [7]. This gives us the opportunity to reconstruct all anatomical details as the resolution provided conforms to the original image. Therefore, the doctor can carefully construct an optimal strategy for a successful surgery, foresee any possible complications and plan in advance how to cope with them.

In our example the heart model produced showed a congenital defect as shown in the image below.

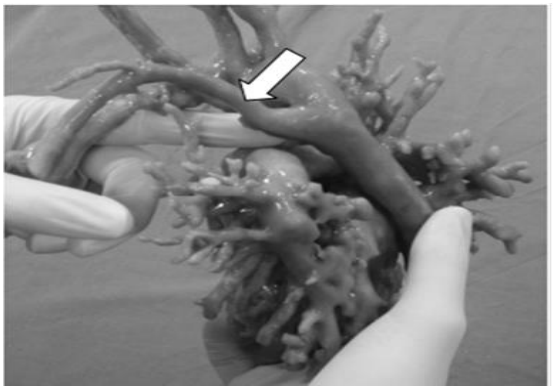

Figure 7. Manufactured model showing a congenital defect. The left subclavian artery (arrow) is abnormally connected to the right descending aorta. This defect is clearly identifiable at the reconstructed 3D model [7].

Apart from denoting any defects of the real organ, the 3D printing object could not possibly serve any other purpose such as organ transplantation.

VII. Conclusion

The contribution of 3d printers in the field of medicine is ground-breaking. With the use of powder printing surgeons are able to assemble three- dimensional heart printed-models from patient-specific data and define the optimal pathway, with regard to cardiac defects under examination. Through the use of bioprinting and syringe extrusion techniques we were able to develop functionally enhanced bionic ears. Moreover, applying powder based printing techniques implants to mend cranial and maxillofacial lesions is a possibility. Whereas, production of inorganic materials like elastomeric hearts or cranial implants is much easier, live organ printing such as a bionic ear capable of being used for implantation in vivo to a human, has been proved to be feasible yet it needs further investigation. In years to come scientists ought to work towards any implementation issues and eliminate critical technological barriers. So forth, it is not arbitrary to predict that in the foreseeable future 3D printing and bioprinting will be used as biomedical research tools as the electron microscope in the 20th century.

References

[1] Ide, A.(2012)the mathematics-of-3d-printing.Retrieved May 16,2013, from http://www.3dfuture.com.au/the-mathematics-of-3d-printing/

[2]Berman, A. "3-D Printing Making the Virtual Real."EDUCAUSE Evolving TechnologiesCommittee. 2007. Web.1 May 2010.

[3]Boland, T., Mironov, V., Gutowska, A., Roth, E. A., & Markwald, R. R. (2003). Cell and organ printing 2: fusion of cell aggregates in three-dimensional gels. [Research Support, U S Gov't, Non-P H S]. Anat Rec A Discov Mol Cell Evol Biol, 272(2), 497-502.

[4]Guillemot, F., Mironov, V., & Nakamura, M. Bioprinting is coming of age: Report from the International Conference on Bioprinting and Biofabrication in Bordeaux (3B'09): Biofabrication. 2010 Mar;2(1):010201.doi: 10.1088/1758-5082/2/1/010201. Epub 2010 Mar 11.

[5]Klammert, U., Gbureck, U., Vorndran, E., Rodiger, J., Meyer-Marcotty, P., & Kubler, A. C. (2010). 3D powder printed calcium phosphate implants for reconstruction of cranial and maxillofacial defects. [Research Support, Non-U S Gov't]. J Craniomaxillofac Surg, 38(8), 565-570.

[6]Mannoor, M. S., Jiang, Z., James, T., Kong, Y. L., Malatesta, K. A., Soboyejo, W. O., Verma, N., Gracias, D. H., & McAlpine, M. C. (2013). 3D Printed Bionic Ears. Nano Letters.

[7]Markert, M., Weber, S., & Lueth, T. C. (2007). A beating heart model 3D printed from specific patient data. Conf Proc IEEE Eng Med Biol Soc, 5, 4472-4475.

[8]Mironov, V., Boland, T., Trusk, T., Forgacs, G., & Markwald, R. R. (2003). Organ printing: computer-aided jet-based 3D tissue engineering. [Research Support, U S Gov't, Non-P H S]. Trends Biotechnol, 21(4), 157-161.

[9]Tsirelson, A. (2011). Analysis 3 Lecture Notes Fubini theorem Available from http://www.tau.ac.il/~tsirel/Courses/Analysis3/lect9.pdf , Last Access June 2013

[10] Zakeri, S. (2007). Math 208 note1 Available from http://www.math.qc.edu/~zakeri/mat208/note1.pdf, Last Access June 2013