Symmetrical Directional Dual-Tree Complex Wavelet Packet Transform

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Abstract— **In this study, a symmetrical directional complex discrete wavelet packet transform, which can be applied directly to the quadrature format signals and has the ability of mapping directional information during decomposition stage, is proposed. With the proposed symmetrical directional complex wavelet packet transform, traditional symmetrical phasing filter technique, which is used for quadrature signal to directional signal conversion, is eliminated and the computational complexity of whole process is reduced. The performance of proposed method is examined in detail using real quadrature embolic signals.**

I. INTRODUCTION

Doppler ultrasound systems that are used in blood flow analysis result in quadrature format signals. In order to derive directional blood flow information, the in-phase and the quadrature-phase components of the quadrature signals must be decoded into the forward and reverse directional signals [1].

In literature, the symmetrical phasing filter technique (Sym-PFT), which is based on Hilbert transform (HT), is the most widely used method in real-time applications for extracting directional signals from the quadrature signals [2]. Traditionally, after obtaining directional signals, time-frequency or time-scale representations based processing methods are applied to these directional signals for further applications such as de-noising and emboli detection [3], [4]. In the scale domain, a complex continuous wavelet transform algorithm which maps the directional information, while doing the analysis, was introduced in [5]. However, for the discrete wavelet transform (DWT) case, an algorithm, which can be applied directly to the in-phase and the quadrature-phase components and has the capability of mapping directional signals in the scale domain during analysis does not exist. Dual tree complex wavelet transform (DTCWT) was proposed as an extension for the DWT. When compared with DWT, the DTCWT has better shift-invariance property and provides better directional selectivity [6]. Despite of its advantages, DTCWT does not provide directional signal decoding during analysis because of its unwanted energy leaks into the negative frequency bands. To overcome this drawback a modified DTCWT is proposed in [7], [8], [9], [10] but it still uses HT filters and delay filters.

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Traditional DWT represents discrete-time signals in dyadic subband decomposition but for specific discrete-time signals such as non-stationary embolic signals, the frequency decomposition provided by the DWT might not be optimal. Embolic signals have more oscillatory behaviour and to analyze these signals, a wavelet transform which has better frequency resolution than the dyadic-DWT is needed. Generalization of the DWT in the discrete wavelet packet transform (DWPT) allows subband analysis without the constraint of dyadic decomposition. The DWPT can perform an adaptive decomposition, which fits the varying signal statistics, of the frequency axis. The DWPT represents a signal in many possible bases and a best decomposition (pruned wavelet packet tree) can be selected from this dictionary according to an optimization criterion [11].

In [12], a dual tree complex wavelet packet transform (DT-CWPT) having approximately shift-invariance and good directional selectivity properties was described. In this complex packet transform, two real DWPTs are employed. The first DWPT can be thought as the real part of the transform while the second DWPT can be thought as the imaginary part of the transform. In DT-CWPT, the wavelet filter banks (FBs) used in imaginary tree are designed according to a certain criterion in order to provide near shift-invariance property. Specifically, imaginary tree wavelet FBs are designed so that their impulse responses are approximately the discrete HTs of those of the real tree wavelet FBs. In order to use DT-CWPT in the analysis of quadrature Doppler signals, firstly forward and reverse directional signals must be obtained and only then two DT-CWPTs can be applied to these two directional signals, Figure 1.

In this study, a symmetrical directional dual tree complex wavelet packet transform (SDDT-CWPT), which eliminates the use of the Sym-PFT step during analysis of quadrature signals and also provides an optimum frequency band representation of embolic signals, is proposed. In this proposed transform, to decrease the computational cost and eliminate Sym-PFT, the HT property of the analysis and synthesis filters of DT-CWPT is used.

II. METHODS

A. Symmetrical Phasing Filter Technique

To understand the proposed directional wavelet transform, the nature of the quadrature signals must be examined in detail. A quadrature Doppler signal can be assumed as a complex signal, in which the in-phase and quadrature-phase parts can be represented as the HT of each other. Mathematically, a discrete quadrature Doppler signal can be

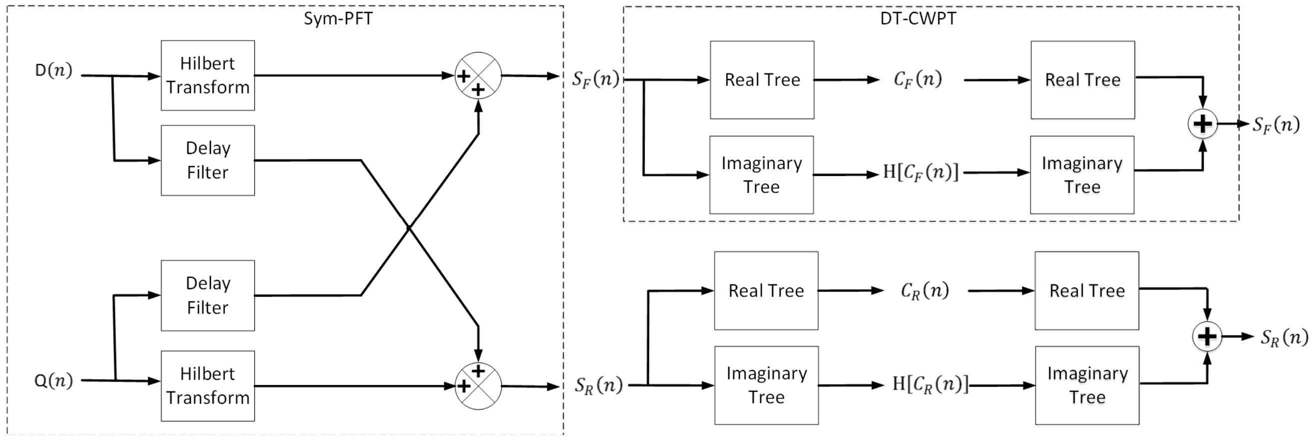

Fig. 1. Structure of a traditional quadrature signal processing system with Sym-PFT plus two DT-CWPTs. $C_F(n)$ and $C_R(n)$ are the coefficients of forward and reverse signals respectively. H[ ] stands for the HT.

modelled as

$$S(n) = Q(n) + jD(n) \tag{1}$$

where *D(n)* is in-phase and *Q(n)* is quadrature-phase components of the signal. In quadrature Doppler signals the directional information is encoded in the phase relationship between *D(n)* and *Q(n)* components. In this respect, *D(n)* and *Q(n)* can also be represented in terms of the directional signals as

$$D(n) = S_F(n) + \mathrm{H}[S_R(n)] \tag{2}$$

$$Q(n) = \mathrm{H}[S_F(n)] + S_R(n) \tag{3}$$

where $S_F(n)$ and $S_R(n)$ represent the forward and reverse flow components respectively and H[ ] stands for the HT.

In the Sym-PFT, which is illustrated in Figure 1, left-side, the HT is applied to both *D(n)* and *Q(n)* components. In practical implementation, two delay filters must also be used to compensate the time delays introduced by the HT filters. So, the outputs of the HT and delay filters constitute a Hilbert pair for each channel [2].

If the *D(n)* and *Q(n)* components are fed into Sym-PFT as inputs, the following results are obtained as the output of the algorithm:

Ignoring the time delays introduced by the digital filters, the HT of *D(n)* is

$$\mathrm{H}[D(n)] = \mathrm{H}[S_F(n)] + \mathrm{H}\Big[\mathrm{H}[S_R(n)]\Big] = \mathrm{H}[S_F(n)] - S_R(n) \tag{4}$$

HT of *Q(n)* is

$$\mathrm{H}[Q(n)] = \mathrm{H}\Big[\mathrm{H}[S_F(n)]\Big] + \mathrm{H}[S_R(n)] = -S_F(n) + \mathrm{H}[S_R(n)] \tag{5}$$

Adding *Q(n)* and H[*D(n)*], and *D(n)* and H[*Q(n)*] yield

$$\mathrm{H}[S_F(n)] + S_R(n) + \mathrm{H}[S_F(n)] - S_R(n) = 2\mathrm{H}[S_F(n)] \tag{6}$$

$$S_F(n) + \mathrm{H}[S_R(n)] - S_F(n) + \mathrm{H}[S_R(n)] = 2\mathrm{H}[S_R(n)] \tag{7}$$

B. Symmetrical Directional Dual Tree Complex Wavelet Packet Transform

It is obvious that the fundamental concept in the Sym-PFT is to shift the phase of in-phase and quadrature-phase components by 90 degrees. The phase shifting operation is performed by the HT filter. In the proposed transform, Symmetrical Directional Dual Tree Complex Wavelet Packet Transform (SDDT-CWPT), the phase shifting operation is attained by the wavelet filters utilised in the imaginary tree in DT-CWPT. This eliminates the HT and delay filter stages in the Sym-PFT and reduces the computational complexity of analyzing quadrature signals with complex wavelet packet transform.

In SDDT-CWPT illustrated in Figure 2, two DT-CWPTs must be used. *Q(n)* and *D(n)* are applied to the each DT-CWPTs respectively. Consequently, for each *Q(n)* and *D(n)* component, the coefficients and their Hilbert transformed versions for a specific subband are obtained for each DT-CWPT channels. Later, in order to obtain the coefficients of directional signals, addition and subtraction operations are employed between resultant coefficients of each DT-CWPT.

If the first DT-CWPTs real tree coefficients for each subband and the imaginary tree coefficients of the second DT-CWPT are added, the Hilbert transformed forward directional coefficients are obtained.

$$C_Q + \mathrm{H}[C_D] = \mathrm{H}[C_F] + C_R + \mathrm{H}[C_F] - C_R = 2\mathrm{H}[C_F] \tag{8}$$

If the first DT-CWPTs imaginary tree coefficients for each subband and the real tree coefficients of the second DT-CWPT are added, the Hilbert transformed reverse directional coefficients are obtained.

$$\mathrm{H}[C_Q] + C_D = -C_F + \mathrm{H}[C_R] + C_F + \mathrm{H}[C_R] = 2\mathrm{H}[C_R] \tag{9}$$

If the first DT-CWPTs imaginary tree coefficients for each subband are subtracted from the real tree coefficients of the second DT-CWPT, forward directional coefficients are obtained.

$$C_D - \mathrm{H}[C_Q] = C_F + \mathrm{H}[C_R] - \Big[-C_F + \mathrm{H}[C_R]\Big] = 2C_F \tag{10}$$

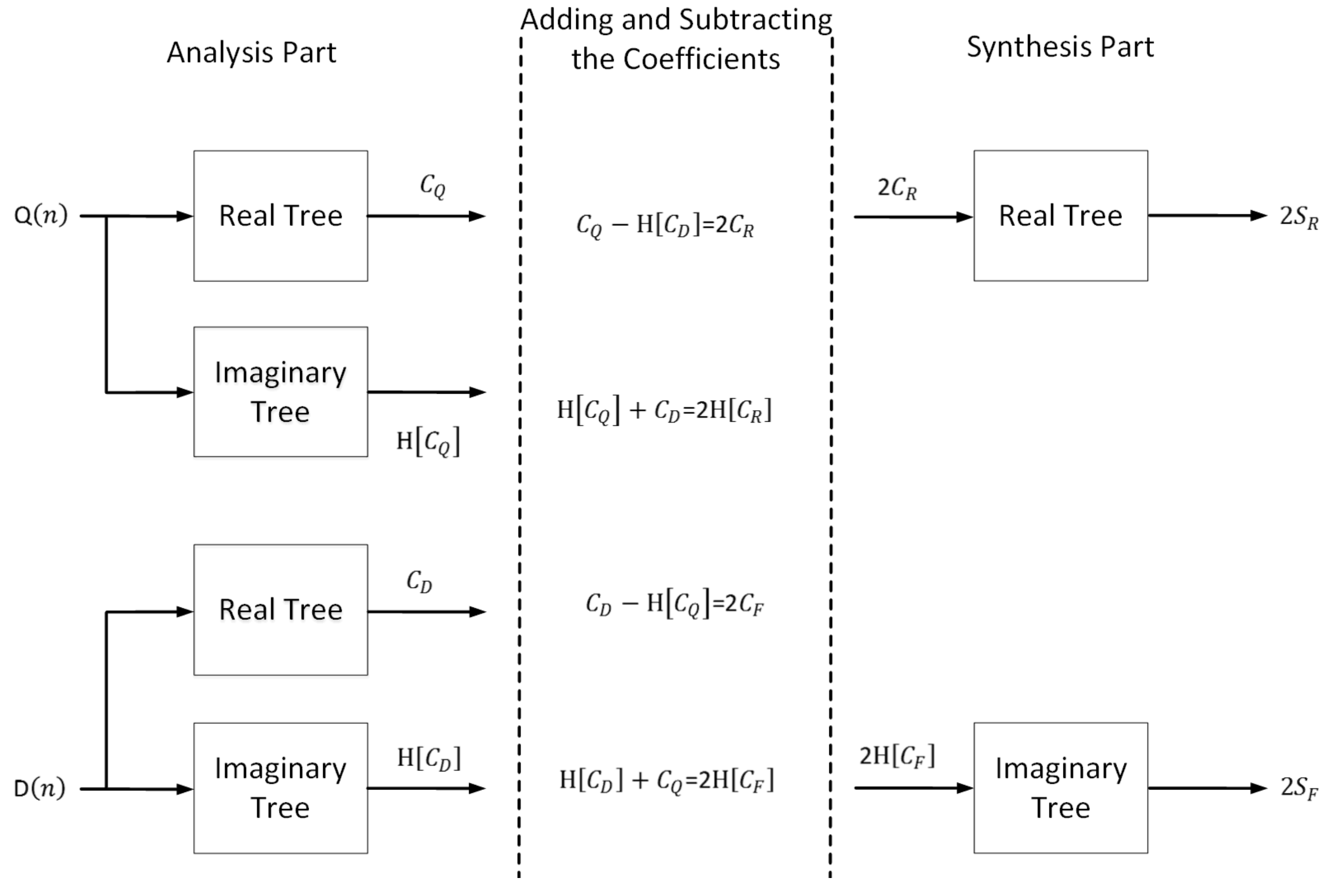

Fig. 2. Structure of Symmetrical Directional Dual Tree Complex Wavelet Packet Transform. C_Q and C_D are the coefficients of *Q(n)* and *D(n)* in the related subband respectively, C_F and C_R are the coefficients of forward and reverse directional signals in the related subband respectively. H[ ] stands for the HT.

If the second DT-CWPTs imaginary tree coefficients for each subband are subtracted from the real tree coefficients of the first DT-CWPT, reverse directional coefficients are obtained.

$$C_Q - \mathrm{H}[C_D] = \mathrm{H}[C_F] + C_R - \Big[\mathrm{H}[C_F] - C_R\Big] = 2C_R \quad (11)$$

where C_Q and C_D are the coefficients of *Q(n)* and *D(n)*; C_F and C_R are the coefficients of forward and reverse directional signals in the related subband respectively.

In the reconstruction part, forward and reverse coefficients are given to the synthesis FBs of the real tree of DT-CWPT resulting in forward and reverse signals at the end of proposed transform.

III. RESULTS

In order to evaluate proposed method's reconstruction performance and also the ability to extract directional information, a quadrature embolic signal is processed with the traditional Sym-PFT and with the SDDT-CWPT. In the SDDT-CWPT, the signal is decomposed and reconstructed for 6 levels without any alterations on coefficients. The full obtained forward signals by using the SDDT-CWPT and the traditional Sym-PFT can be seen in the Figure 3. As it can be seen from the Figure 3, by using the SDDT-CWPT, directional signals can be obtained accurately at the end of the proposed transform.

IV. CONCLUSIONS

In this study, a symmetrical directional complex wavelet packet transform, which can be applied directly to quadrature signals and have the ability of extracting directional information during analysis, is introduced. With this proposed directional transform, the traditional Sym-PFT step, which is used for extracting directional signals prior to wavelet analysis, is eliminated. In order to measure the performance of proposed method, a quadrature Doppler signal is processed with both the classical Sym-PFT and SDDT-CWPT. The results verify that with the proposed method, directional information can be obtained accurately at the end of proposed transform. In the future, this proposed transform can be employed in an embolic signal detection algorithm in order to obtain sparse representations of emboli information in decomposed subbands.

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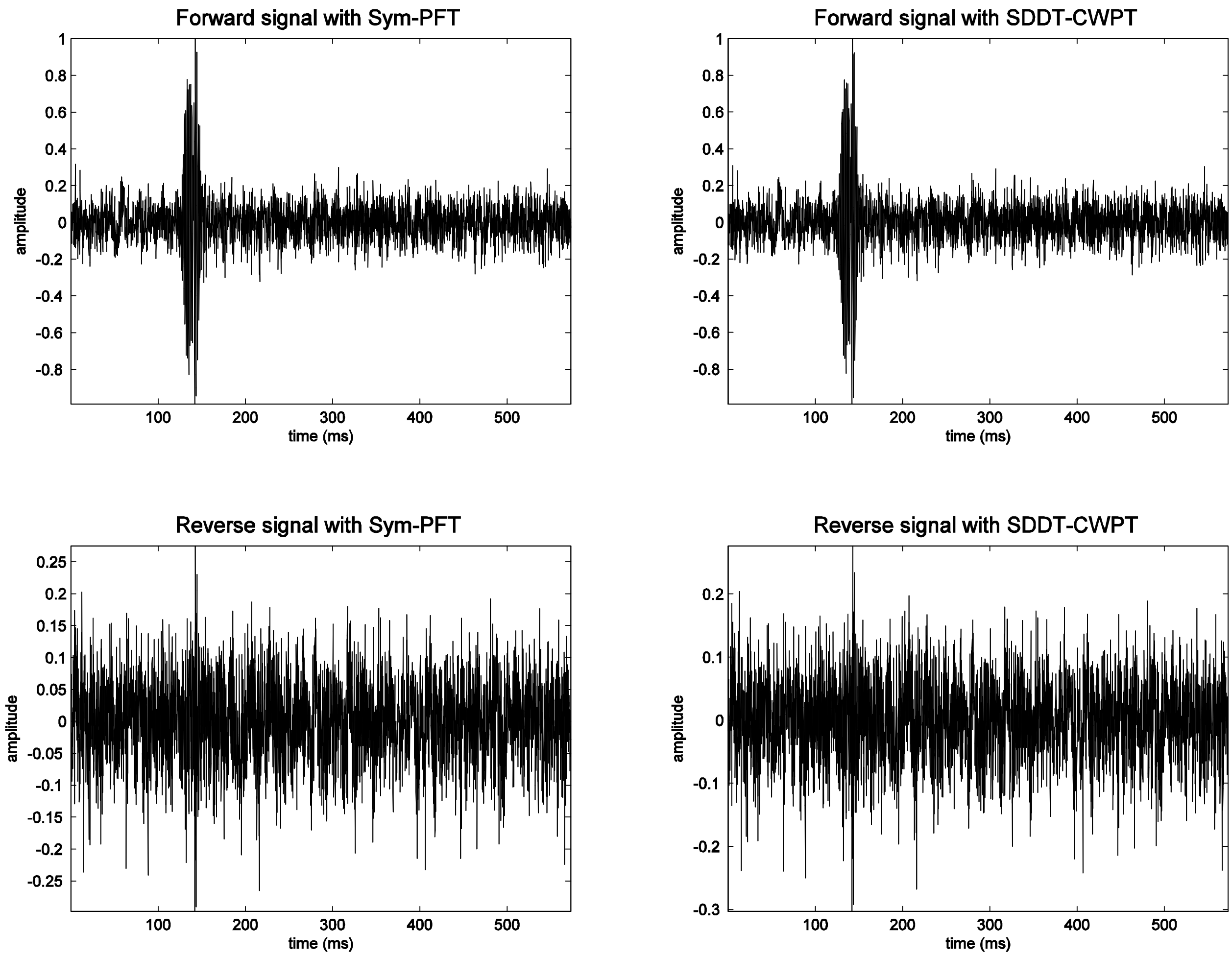

Fig. 3. Output of proposed SDDT-CWPT and Sym-PFT for a quadrature embolic signal.

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