Directional Dual-Tree Rational-Dilation Complex Wavelet Transform

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***Abstract*— Dyadic discrete wavelet transform (DWT) has been used successfully in processing signals having non-oscillatory transient behaviour. However, due to the low Q-factor property of their wavelet atoms, the dyadic DWT is less effective in processing oscillatory signals such as embolic signals (ESs). ESs are extracted from quadrature Doppler signals, which are the output of Doppler ultrasound systems. In order to process ESs, firstly, a pre-processing operation known as phase filtering for obtaining directional signals from quadrature Doppler signals must be employed. Only then, wavelet based methods can be applied to these directional signals for further analysis. In this study, a directional dual-tree rational-dilation complex wavelet transform, which can be applied directly to quadrature signals and has the ability of extracting directional information during analysis, is introduced.**

I. INTRODUCTION

Dyadic discrete wavelet transform (dyadic-DWT), in which the resolution is doubled from each scale to the next scale, is a very effective tool for processing piecewise smooth signals and used as a popular transform in various signal and image processing applications. However, due to its poor frequency resolution and severe frequency aliasing drawbacks, the dyadic-DWT is less effective in processing signals having more oscillatory behaviour, such as speech, EEG, etc. These type of signals are quasi-periodic over short-time intervals. In analyzing these signals, a wavelet transform having better frequency resolution than the dyadic-DWT is needed. Additionally, the dyadic-DWT lacks of shift invariance property causing considerable distortions in the coefficients resulting in at the end of decomposition stage of wavelet analysis when the input signal is time-shifted.

In order to overcome the drawbacks of the dyadic-DWT in processing oscillatory signals, various overcomplete WTs such as the dual-tree complex wavelet transform and the double density WT have been proposed [1], [2]. Many of these WTs have the capability of increasing only the temporal resolution with limited range of redundancy factors. However in [3], an overcomplete wavelet transform, which is based on rational (non-dyadic) dilation factors, are proposed. This transform is based on a frequency domain design and the implementation is based on the fast Fourier transform. Additionally, the transform is appropriate for the discrete-time signals, approximately shift invariant, and easily invertible. In this transform, various Q-factors and redundancy factors can be attained by changing the dilation factor. Due to its being overcomplete and rational dilation property, this transform is called overcomplete rational dilation wavelet transform (RADWT).

Quadrature signals, which are composed of the in-phase and the quadrature-phase components, are obtained at the detection stage of the systems employing quadrature demodulation technique. In biomedical engineering, Doppler ultrasound systems that are used in blood flow analysis have also quadrature format outputs and the outputs of these systems are frequently used in embolic signal detection. Emboli are small travelling clots which can originate in an artery of the body, traveling up the arterial tree to the brain until it lodges in a blood vessel. The presence of emboli in blood circulation can be the main factor of stroke, which may cause permanent damage or even death. In order to prevent stroke, asymptomatic circulating cerebral emboli can be detected by analyzing quadrature Doppler ultrasound signals.

However, to process quadrature Doppler ultrasound signals, firstly the in-phase and the quadrature-phase components must be decoded into the forward and reverse direction components of blood flow. In literature, the phasing filter technique (PFT) is the most widely used method to obtain directional signals. In the PFT, Hilbert transform (HT) of one of the in-phase and the quadrature-phase components must be taken to introduce 90 degree phase shift [4], [5]. After obtaining directional signals, Fourier transform and wavelet transform based algorithms are applied for further analysis.

In literature, a complex continuous wavelet transform algorithm which is applied to directly quadrature signals and maps the directional information, while doing the analysis, was introduced in [6]. In the DWT case, a transform which can be applied directly to the in-phase and the quadrature-phase components and has the capability of mapping directional signals in the scale domain during analysis does not exist. To achieve such a property, the DWT must be a complex transform with only one-side frequency spectrum. In [2] a dual tree complex wavelet transform (DTCWT) was proposed but because of its energy leakages into its negative frequency bands, it cannot achieve desired one-sided frequency spectrum property. In [7], [8], [9], [10] a modified DTCWT is proposed. However, in this transform still an additional HT filter and a delay filter (used in digital circuits to compensate the time delay that occurred because of the digital HT filter) must be used. Additionally it is still a dyadic-DWT.

Embolic signals are usually described as amplitude-

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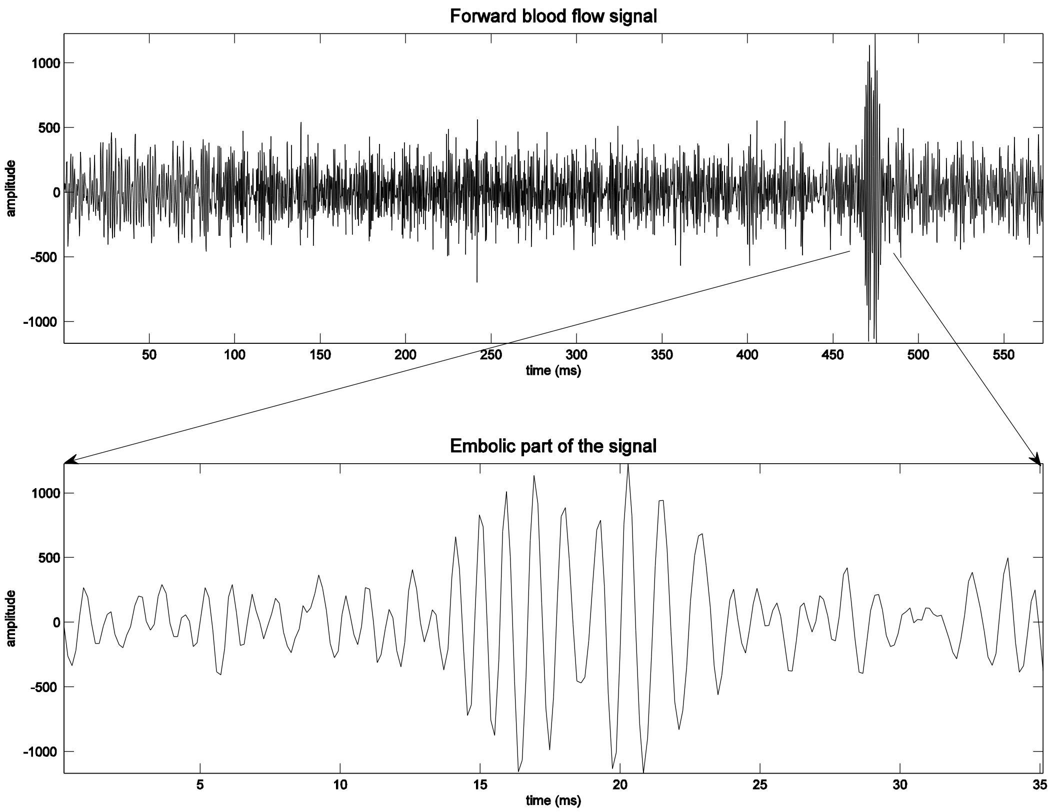

Fig. 1. A forward blood flow signal (above) and zoomed version of emboli part (below).

modulated sine waves which have very short duration varying between 2 and 100 ms. Some examples are illustrated in Figure 1. In frequency domain, embolic signals result in an increase in intensity that is focused on a small band of frequencies (behave as a narrow-band signal) in the Doppler spectrum, resulting in a bell-shaped distribution. In [11] the dyadic DWT was applied to directional blood-flow signals in order to detect embolies. However, in order to achieve better detection accuracies for embolic signals, RADWT, whose Q-factor and redundancy can be adjustable, can be utilized. By tuning the Q-factor (increasing the Q-factor) of the RADWT in decomposition, an optimum oscillatory wavelet behaviour can be set for an embolic signal and a more compact energy distribution of the coefficients can be obtained. In [12] a modified version of RADWT, a dual-tree rational-dilation wavelet transform (DT-RADWT) was introduced that inherits the good frequency resolution and constant-Q property of the RADWT and whose atoms form quadrature pairs.

In this study, we introduce a directional DT-RADWT (DDT-RADWT) that extends the classical DT-RADWT. The DDT-RADWT is complex transform which can be directly applied to quadrature signals. Its frequency resolution can be changed by tuning the Q-factor of wavelets according to the behaviour of analyzed signal.

II. METHODS

A. Quadrature Doppler Signals and Phase Filtering Technique

A quadrature Doppler signal can be assumed as a complex signal, in which the in-phase and quadrature-phase parts can be represented as the HT of each other. Mathematically, a discrete quadrature Doppler signal can be modelled as

$$s(n) = Q(n) + jD(n) \tag{1}$$

where *D(n)* is in-phase and *Q(n)* is quadrature-phase components of the signal.

In the PFT two types of filters are used for decoding directional blood flow signals from quadrature format. The first filter is the HT filter and the second one is delay filter introducing a time delay that equals to the time delay introduced by the finite impulse response type HT filter. The structure of the algorithm can be seen in Figure 2.

B. Rational Dilation Wavelet Transform

Due to the low Q-factor property of their subbands (low oscillatory nature of wavelet bases), dyadic DWTs are less effective in processing oscillatory signals such as embolic signals. In this respect, RADWT, which is a fully discrete, easily invertible, energy preserving, and approximately shift-invariant, can be employed. The RADWT provides ability to

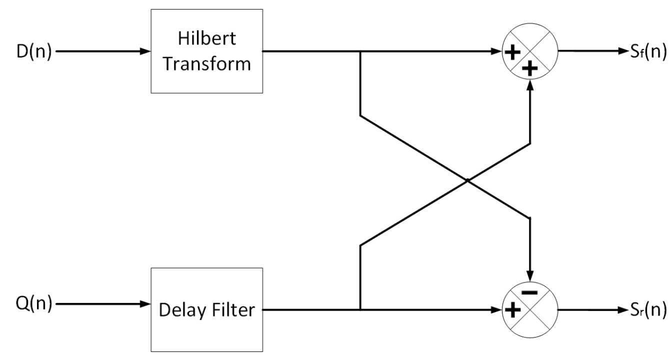

Fig. 2. Implementation of the PFT.

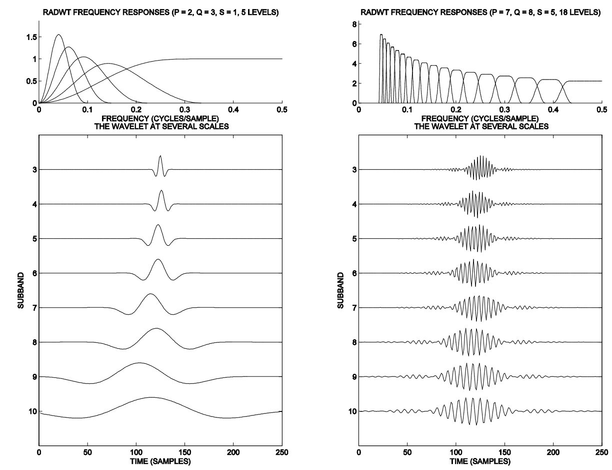

Fig. 3. Frequency response of decomposed subbands and associated wavelets with low Q-factor (left) and high Q-factor (right).

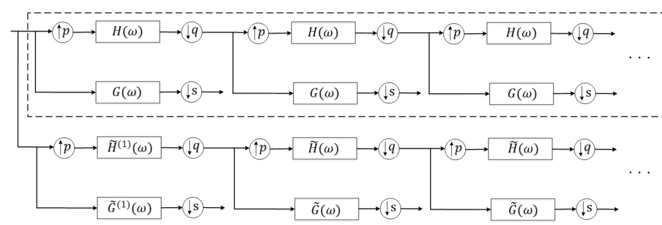

Fig. 4. The decomposition stage of the DT-RADWT.

the user to adjust the Q-factor of wavelet bases. It can be used for high Q-factor analysis or the same low Q-factor analysis as the widely used dyadic DWT. In RADWT, by changing the transform parameters not only the frequency partition manner, but also the time domain oscillatory nature of the wavelet functions are controlled. Similar to the classical DWT implemented using Mallats tree-structure filterbank, the RADWT is also implemented through an iterated two-channel filter-bank (FB). The Q-factor of the wavelet transform depends on the parameters p, q and s. Instead of being based on integer dilations, the dilation factor of the transform is q/p where the numbers q and p are co-prime and satisfy $q > p$. In order to obtain higher frequency resolution, for q and p values, following conditions must be satisfied; $1 < q/p < 2$, q/p ratio must be close to 1, and $s > 1$. When the s is set to 1, a classic dyadic DWT with low Q-factor is obtained. In Figure 3, in upper left and right parts, the frequency decomposition of the RADWT with a low Q-factor (5 levels) and high Q-factor (18 levels) can be seen. Additionally, in lower left and right parts, associated wavelets from levels 3 to 10 with a low Q-factor and high Q-factor can be seen respectively. As it can be seen, when the Q-factor is increased, in time domain, the wavelets obtained in subbands become more similar to embolic signals and their frequency responses become more frequency selective.

C. Dual-Tree Rational-Dilation Complex Wavelet Transform

In [12], the dual-tree rational-dilation complex wavelet transform (DT-RADWT), which employs quadrature pairs of time-frequency atoms similar to the short time Fourier transform and the dyadic DTCWT for oscillatory signal processing was introduced. DT-RADWT is a constant-Q transform, a property lacked by the short-time Fourier transform, which in turn makes the introduced transform more suitable for models that depend on scale. Additionally, In the DT-RADWT, the frequency resolution can be adjusted and high Q-factor, a property lacked by the dyadic DTCWT, can be attained. This property makes the DT-RADWT more suitable for processing oscillatory signals such as embolic signals [12]. DT-RADWT is realized by using two wavelet trees, one is the real tree and the other is imaginary tree, operating in parallel on the same input. In DT-RADWT, the second wavelet FBs (FBs of imaginary tree) are designed so that their impulse responses are approximately the discrete HTs of those of the first wavelet FBs (FBs of real tree). Then, to process quadrature signals, this capability of taking HT property of the imaginary tree in DT-RADWT, can be used to obtain 90 degrees phase shift effect of the classical PFT. The structure of DT-RADWT is illustrated in Figure 4. In the dashed box, real part of the DT-RADWT can be seen.

D. Directional Dual-Tree Rational-Dilation Complex Wavelet Transform

Conventionally, in order to utilize RADWT in the analysis of the quadrature Doppler signals, firstly, directional signals (forward signal and reverse signal) must be obtained by using the PFT. Only then, the RADWT can be applied with the aim of emboli detection. This procedure increases the computational cost of the processing system. However, it is possible to reduce the computational cost of the processing system by utilizing the HT property of the analysis and synthesis filters in the imaginary tree of the DT-RADWT. In the proposed method (Directional DT-RADWT), two modifications are made to the conventional DT-RADWT, as illustrated in Figure 5. At the decomposition stage, only the quadrature-phase part (*Q(n)*) is applied to the real tree and only the in-phase part (*D(n)*) is applied to the imaginary tree. Eventually, as an output of real tree, coefficients of *Q(n)* and as an output of imaginary tree, HT'ed coefficients of *D(n)* are

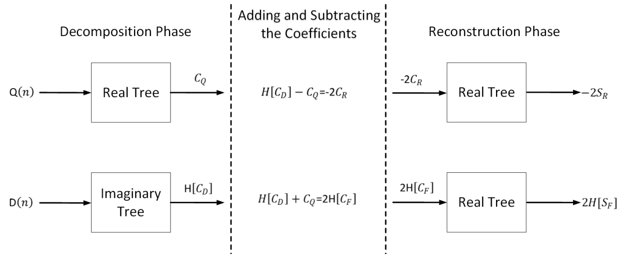

Fig. 5. The structure of proposed Directional DT-RADWT.

obtained. Later, the coefficients of *Q(n)* are subtracted from the HT'ed coefficients of *D(n)* resulting in the coefficients of reverse signal. Likewise, HT'ed coefficients of *D(n)* are added to the coefficients of *Q(n)* resulting in the HT'ed coefficients of forward signal. In the decomposition stage, the coefficients of forward and reverse signal are given to synthesis filters of the real tree of DT-RADWT resulting in the forward and reverse directional signals.

III. RESULTS

In order to evaluate proposed method's reconstruction performance and also the ability to extract directional information, a quadrature embolic signal is processed with the traditional PFT and with the directional DT-RADWT (DDT-RADWT). In the DDT-RADWT, the signal is decomposed and reconstructed for 20 levels without any alterations on coefficients. The full obtained forward signals by using the DDT-RADWT and the PFT can be seen in the two upper rows of Figure 6. Additionally, to see the reconstruction success on emboli parts, a zoomed version of embolic signals obtained with the PFT and DDT-RADWT can be seen in the lower third row of Figure 6. As it can be seen from the figure, by using the DDT-RADWT, directional signals can be obtained accurately at the end of the proposed transform.

IV. CONCLUSIONS

In this study, a DDT-RADWT, which can be applied directly to quadrature signals and have the ability of extracting directional information during analysis, is introduced. With the proposed directional transform, the traditional PFT step, which is used for extracting directional signals prior to wavelet analysis, is eliminated. In the DDT-RADWT, frequency resolution of wavelet atoms can be changed by tuning their Q-factor according to the behaviour of analyzed signal. Additionally, the proposed transform has near shift invariance property, which is very important in processing quadrature Doppler signals due to the phase relationship between their in-phase and quadrature-phase components. In order to measure the performance of the proposed method, a quadrature Doppler signal is processed with both the classical PFT and DDT-RADWT. The results verify that with the DDT-RADWT, directional information can be obtained accurately at the end of the transform without any distortion in the emboli information. In the future, DDT-RADWT can be employed in an embolic signal detection algorithm in order to obtain sparse representations of emboli information in decomposed subbands.

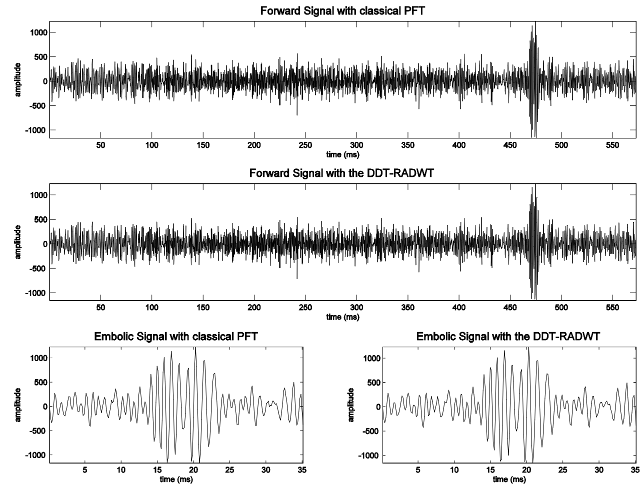

Fig. 6. In the upper two rows, a forward blood flow signal obtained with the classical PFT and proposed directional DT-RADWT is illustrated. In the third row, embolic signal parts obtained with with the classical PFT and proposed directional DT-RADWT are illustrated.

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