Low Dose PET Reconstruction with Total Variation Regularization

Chenye Wang, Zhenghui Hu, Pengcheng Shi and Huafeng Liu*

***Abstract*— Low dose positron emission tomography(PET) reconstruction remains a challenging issue for statistical PET reconstruction methods due to the low SNR of data. Due to the ill-conditioning of image reconstruction, proper prior knowledge should be incorporated to constrain the reconstruction. Since PET images are piecewise smoothing, we propose the total variational (TV) minimization based algorithm for low dose PET imaging. The fundamental power of this strategy rests with the edge locations of important image features tend to be preserved thanks to TV regularization. In addition, a new computational method have been employed with improved computational speed and robustness. Experimental results on Monte Carlo simulations demonstrate its superior performance.**

I. INTRODUCTION

Positron emission tomography (PET) [1] is a nuclear medicine, functional imaging technique that produces a three-dimensional image of functional processes in the body. The system detects pairs of gamma rays emitted by a positron-emitting radionuclide (tracer), which is introduced into the body on a biologically active molecule.

The realization of low dose PET is a promising task. It means the reduction of radiation in PET scanning. Less radioactive tracer can cause the reduction of the cost and make PET scanning healthier for both patients and staff. Further more, there are close clinical injection rule in some certain circumstances, such as pediatric PET scanning. Common adult injection rules prescribe either a dose proportional to weight or a fixed dose. If better quality than in average-adult studies does not justify the associated dose burden, attractive options are to reduce scan time, reduce dose, or any combination of the two [2], [3].

Iterative statistical methods have been the primary focus of many recent efforts, including notable examples such as maximum likelihood (ML)-expectation maximization (EM) [4], [5], maximum a posteriori (MAP) [6], [7], and penalized weighted lease-squares (PWLS) [8]. Nevertheless, these iterative statistical methods still have certain drawbacks especially for low dose case. First, they face difficulties in handling low SNR data. Secondly, after random events substraction, correction for scanner sensitivity and dead time, attenuation and scatter corrections, the actual statistical model of measured data violates the statistical assumption. For any statistical image reconstruction framework, it is clear that accurate statistical measurement model play an essential role in achieving good reconstruction [9]. Without additional constraints, it can be expected that the statistical methods could not give satisfied results. In order to overcome these issues, other constraints are needed.

One important PET image feature are the edges: these are places in an image where there is a sharp change in image properties, which happens for instance at object (i.e. tumor) boundaries. Unlike statistically-based methods, total variational(TV) is incorporated to provide edge-preserving guidance for the reconstruction. In this research, the reconstruction problem is transformed to a minimization problem of augmented Lagrangian function. The data fitting terms and the regularization term are included in the function. And the alternating minimization involving 2D shrinkage-like formula and the steepest descent method is applied to solve the minimization problem.

We realize that several efforts in PET image reconstruction are of relevance to our work. Nonlinear variational method have been proposed into the reconstruction process to make an efficient use of a-priori information and to attain improved imaging results [10]. Poisson based negative log likelihood function is utilized in the algorithm. A total variation based EM algorithm has been proposed. It enhanced object edges far better than the EM-method [11], but still with limited success due to strong computational difficulties in the minimization with total variation as a regularization function.

The advantages of TV minimization stem from the property that it can recover not only sparse images, but also dense staircase piecewise constant images. In other words, TV regularization would succeed when the gradient of the underlying image is sparse. It is the circumstance of low dose PET reconstruction. The method proposed – Alternating minimization of augmented Lagrangian is to be called AMAL for short.

II. METHOD

A. PET imaging model

PET acquired data are organized in a series of parallel slices that can be reconstructed independently. And every slice of raw data collected by a PET scanner is a list of coincidence events representing near-simultaneous detection of annihilation photons by a pair of detectors. Each coincidence event represents a line in space connecting the two detectors along which the positron emission occurred (the line of response (LOR)). The raw data from PET is organized in sinogram.

Therefore, PET image reconstruction problems are specific cases of the following general inverse problem: find an

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estimate of radioactive activity map u from a measurement b by

$$b = Au + noise \quad (1)$$

In the process of PET imaging, u_i is the radioisotope concentration in the ith pixel, A is the system matrix that describes the tomographic geometry and the physical factors. Under the circumstance of low dose PET, the number of coincidence events in b is much smaller than the normal PET. It makes the accurate reconstruction a challenge issue.

The image u to be reconstructed is piecewise smoothing. And the edges in the image describe the structures of tissues, which is an important feature. Some hybrid technologies like PET-CT and PET-MRI have been developed to obtain precise structures of tissues.

B. Problem formulation

Total variation regularization is a recently emerging image processing techniques using partial differential equations which has been shown to be very successful in many image processing applications. It is based on the principle that images with excessive and possibly spurious detail have high total variation. According to this principle, reducing the total variation of the image subject to it being a close match to the original image, removes unwanted detail whilst preserving important details [12].

Total variation regularizer was first proposed for image denoising by Rudin, Osher and Fatemi in [13], and then extended to image deblurring in [14]. In comparison to the well known Tikhonov-like regularizers, TV regularizers can better preserve sharp edges or object boundaries that are usually the most important features to recover.

Thus we apply the TV regularization in the reconstruction. The problem is formulated as:

$$\min_u \{TV(u) + \frac{\mu}{2}\|Au - b\|_2^2\} \quad (2)$$

where μ is a parameter and $TV(u)$ is defined as $\sum_i \|D_i u\|$, the sum of the discrete gradient of activity map u of every pixel i .

C. Solution

It is hard to solve the minimization problem (2) directly. So we introduce a new variable w. At each pixel an auxiliary variable w_i is introduced to transfer $D_i u$ out of the nondifferentiable term $\|.\|$. And problem (2) is transformed to problem (3)

$$\min_{w_i, u} \sum_i \|w_i\|, \quad s.t. \quad Au = b \text{ and } D_i u = w_i \text{ for all } i \quad (3)$$

To deal with the constrains, we transform the problem (3) to a minimization problem of augmented Lagrangian function, making the constrained problem an equivalent unconstrained problem. And the corresponding augmented Lagrangian function of problem (3) is

$$\begin{aligned} L_A(w_i, u) = \sum_i (\|w_i\| - \upsilon_i^T (D_i u - w_i) \\ + \frac{\beta_i}{2}\|D_i u - w_i\|_2^2) - \lambda^T (Au - b) + \frac{\mu}{2}\|Au - b\|_2^2 \end{aligned} \quad (4)$$

here the first term is the regularization term. and the difference between w_i and $D_i u$ is penalized. The second and forth terms are linear penalty terms. And the third and fifth terms are quadratic penalty terms. These penalty terms ensure the reconstruction fits the basic model (1) well. υ_i, β_i, λ and μ are the multipliers of the four penalty terms. In order to make the result of every term a number rather than a matrix, transpose of υ and λ are utilized in the equation (4). The iterative algorithm of the alternating minimization is applied to solve the minimizers: u and w_i.

Let u_k and $w_{i,k}$ represent the true minimizers of equation (4) at kth iteration. $w_{i,k+1}$ can be attained by

$$\begin{aligned} \min_{w_i} L_A(w_i, u_k) = \sum_i (\|w_i\| - \upsilon_i^T (D_i u_k - w_i) \\ + \frac{\beta_i}{2}\|D_i u_k - w_i\|_2^2) - \lambda^T (Au_k - b) + \frac{\mu}{2}\|Au_k - b\|_2^2 \end{aligned} \quad (5)$$

And it is equivalent to the problem follows:

$$\min_{w_i} \sum_i (\|w_i\| - \upsilon_i^T (D_i u - w_i) + \frac{\beta_i}{2}\|D_i u - w_i\|_2^2) \quad (6)$$

For given $\beta > 0$,the minimizer of equation (6) is given by the 2D shrinkage-like formula. So the we can get $w_{i,k+1}$ by

$$w_{i,k+1} = \max\{\|D_i u_k - \frac{\upsilon_i}{\beta_i}\| - \frac{1}{\beta_i}, 0\} \frac{(D_i u_k - \frac{\upsilon_i}{\beta_i})}{\|D_i u_k - \frac{\upsilon_i}{\beta_i}\|} \quad (7)$$

With $w_{i,k+1}$, we can achieve u_{k+1} by

$$\begin{aligned} \min_u L_A(w_{i,k+1}, u) = \sum_i (\|w_{i,k+1}\| - \upsilon_i^T (D_i u_k - w_{i,k+1}) \\ + \frac{\beta_i}{2}\|D_i u_k - w_{i,k+1}\|_2^2) - \lambda^T (Au_k - b) + \frac{\mu}{2}\|Au_k - b\|_2^2 \end{aligned} \quad (8)$$

Similar to the method to achieve $w_{i,k+1}$, this subproblem is equivalent to the problem follows:

$$\begin{aligned} \min_u \sum_i (-\upsilon_i^T (D_i u_k - w_{i,k+1}) + \frac{\beta_i}{2}\|D_i u_k - w_{i,k+1}\|_2^2) \\ -\lambda^T (Au_k - b) + \frac{\mu}{2}\|Au_k - b\|_2^2 \end{aligned} \quad (9)$$

Its gradient is

$$\begin{aligned} d_k(u) = \sum_i (\beta_i D_i^T (-D_i u - w_{i,k+1}) - D_i^T \upsilon_i) \\ + \mu A^T (Au - b) - A^T \lambda \end{aligned} \quad (10)$$

here u_{k+1} can be achieved by forcing $d_k(u) = 0$. The problem is solved iteratively by the steepest descent method. The iterative equation is :

$$u_{k+1} = u_k - \alpha_k d_k u_k \quad (11)$$

What left to do next is choosing α. Barzilai and Borwein [15] suggested an aggressive manner to choose step length for the steepest descent method, which is called the BB step or BB method. This method is applied to choose α:

$$\alpha_k = \frac{(u_k - u_{k-1})^T (u_k - u_{k-1})}{(u_k - u_{k-1})^T (d_k(u_k) - d_k(u_{k-1}))} \quad (12)$$

D. Parameters

There are several parameters in the algorithm, and μ is the most important one. It determines the weight of data fitting term. Therefore, to get the best performance, the value of μ should be set according to the noise level in the observation b . For example, the higher the noise level is, the smaller μ should be. The value of β also affects the performance of AMAL, but it is much less important than μ. We decide β by trying with values from 2^4 up to 2^{13} . For the initialization, we set initial μ_0 and β_0 much smaller than μ and β, respectively.

υ_i and λ were initialized to be $\mathbf{0}$ and updated as long as equation (4) is minimized at each iteration. According to formula proposed by Hestenes [16] and Powell [17], the update formulas of multipliers follow

$$\upsilon_{i,k+1} = \upsilon_{i,k} - \beta_{i,k}(D_i u_k - w_{i,k}) \tag{13}$$

$$\lambda_{k+1} = \lambda_k - \mu_k(Au_k - b) \tag{14}$$

The stopping tolerance determines the solution accuracy. The smaller value results in a longer elapsed time and usually a better solution quality. If the observation is noisy or the problem is large-scale,stopping tolerance $= 1 * 10^{-3}$ might be sufficient.

III. Experiments and Results

The synthetic emission phantom with known radioactivity concentrations is used in the GATE platform, as shown in Fig. 1. The resolution of the original image is 64 by 64 pixels. 720 projections over 180 degrees are simulated. Five sinograms are generated with the total number of photon counts in the reconstruction plane set to be $5*10^5$, $1*10^6$, $3*10^6$, $6*10^6$ and $9*10^6$ respectively. A mask is calculated to eliminate the pixels with 0 value in sinograms. And the same mask is used on system matrix in order to make the dimensions match. We compare our proposed method with ML-EM algorithm. The relative errors bias and variance are calculated through $bias = \frac{1}{n}\sum_{i=1}^{n}(\hat{u}_i - u_i)$ and $variance = \frac{1}{n}\sum_{i=1}^{n}(\hat{u}_i - u_i)^2$. The analysis results are summarized in Tab I. Images reconstructed by the two algorithms (EM and AMAL) with five different measurements are shown in Fig. 2, with the selected vertical profiles plotted versus the corresponding pixel positions shown in Fig. 3.

These quantitative results and figures illustrate that the images reconstructed by AMAL are visually more similar to the ground truths with sharper edges. Both the bias and variance of the images reconstructed by AMAL are lower than the ones reconstructed by EM. The profile shows the value of the 38th row of each image. And the result in Fig. 3 shows that line of AMAL fits the ground truth better than one of EM. Furthermore, Tab I shows that AMAL is also efficient under the circumstance of low dose PET imaging.

Fig. 1. The Hoffman brain phantom.

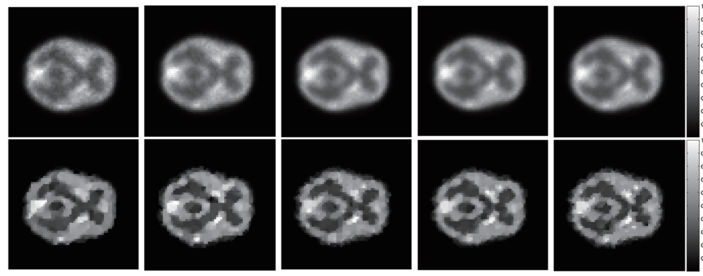

Fig. 2. Images reconstructed by EM(the first row), images reconstructed by AMAL(the second row)from the data with different number of counts . From left to right: $5*10^5$, $1*10^6$, $3*10^6$, $6*10^6$ and $9*10^6$

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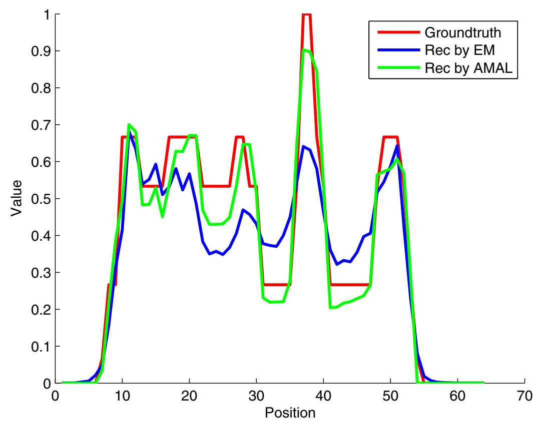
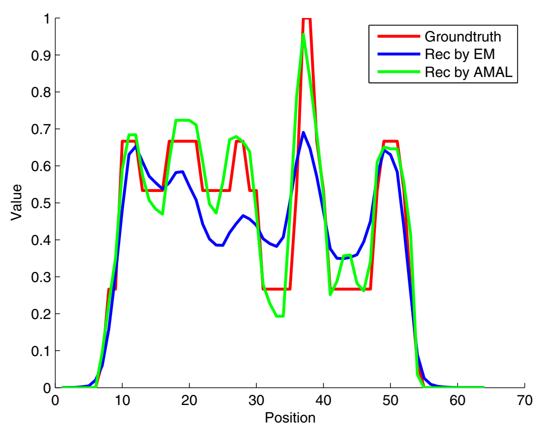
TABLE I

QUANTITATIVE ANALYSIS OF THE RECONSTRUCTIONS FROM FIVE COUNTING LEVEL DATA BY EM AND AMAL

Counting Level	$5*10^5$	$1*10^6$	$3*10^6$	$6*10^6$	$9*10^6$
EM Bias	0.0357	0.0334	0.0329	0.0329	0.0330
EM Variance	0.0134	0.0123	0.0116	0.0116	0.0116
AMAL Bias	0.0319	0.0281	0.0275	0.0263	0.0248
AMAL Variance	0.0119	0.0102	0.0089	0.0078	0.0069



(a) Profile of $5*10^5$



(b) Profile of $9*10^6$

Fig. 3. Profiles of reconstructions from different data. X axis : the vertical position of pixel; Y axis : value of the pixel.

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