Power spectrum of the rectified EMG: influence of motor unit action potential shapes

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***Abstract*— The rectification of EMG signals is a pre-processing method widely used for inferring neural connectivity by coherence analysis. The assumption for the use of this non-linear operator is that it enhances the neural information in the signal, i.e. the motor unit spike trains. However, because of non-linearity, it is difficult to predict the effect of rectification on the EMG power spectrum. In this study, we analyze the influence of the motor unit action potential properties on spectral content of the rectified EMG. The results show that changes in the action potential waveforms have a strong influence on the rectified EMG power spectrum, with an effect on estimated coherence functions. Knowledge on the properties of action potentials may be necessary for properly comparing the rectified EMG power spectrum across conditions.**

I. Introduction

The surface EMG is often rectified before spectral analysis. This procedure is justified by the claim that rectification enhances the neural information in the signal, i.e. the motor unit spike trains [1]. Rectification is specifically suggested when computing EEG-EMG or EMG-EMG coherence measures [2]. However, rectification, which is non-linear, generates a certain degree of distortion in the processed signal [3-5]. For example, if the level of muscle activation in the signal (number of active fibers) is relatively high, the resulting rectified EMG may carry a substantial alteration of the original neural information because of amplitude cancellation [6]. In addition to this effect, the rectification of the motor unit action potentials also changes the filtering of the neural signal. However, this latter effect has never been described in detail. In this study, we describe the influence of the motor unit action potential shapes on the rectified EMG signal. We provide an analytical derivation of the rectified signal for a simplified model of motor unit action potential. The results further clarify the effect of rectification on the EMG spectral properties.

II. Methods

A. Theory

In this section, we describe analytically the influence of the motor unit action potentials on the rectified EMG signal. The EMG signal $y(t)$ is the sum of the convolutions between the spike trains generated by the N active motor neurons and the corresponding motor unit action potentials:

$$y(t) = \sum_{i=1}^{N} s_i(t) * p_i(t) \qquad (1)$$

where $s_i(t)$ is the i-th spike train and $p_i(t)$ the i-th motor unit action potential. Because the motor unit action potentials have duration (10-20 ms [7]) smaller than the inter-spike intervals in normal conditions (25-33 ms [8]), the rectified EMG signal $|y(t)|$ can be expressed as:

$$\left|y(t)\right| = \sum_{i=1}^{N} s_i(t) * \left|p_i(t)\right| - c(t) = z(t) - c(t) \qquad (2)$$

where $z(t)$ and $c(t)$ are the no-cancellation and the cancellation signals respectively [6]. However, during fast contractions, the inter-spike intervals can reach values of 10 ms [9]. In these extreme conditions, Eq. 2 provides only an approximation. The first term of Eq. 2 can be seen as the linear part, since it is simply the convolution between the original spike trains and the rectified motor unit action potentials. The second term represents the distortion due to non-linearity. Since the second term has been previously investigated [6], we now focus on the first term, i.e. on the filtering of the neural information by the rectified action potentials. It is important to notice that the power spectrum of the rectified EMG contains cross-terms due to the interaction of the non-cancellation and cancellation components. However, in the following we are mainly interested in the influence of the motor unit action potentials on the non-cancellation term assuming that the contribution of cancellation is relatively weak (e.g. low to moderate contraction levels).

To study the filtering effect in the first term of Eq. (2), we will model the action potential shape with an analytical function for which we will compute the Fourier transform with and without rectification. For this purpose, the waveform of a bipolar motor unit action potential is approximated by a Hermite-Rodriguez (HR) function of the first order, so that its rectified version is:

$$\left|w_{\lambda,1}(t)\right| = \frac{2}{\sqrt{2\pi}} \frac{|t|}{\lambda^2} e^{-t^2/\lambda^2} \qquad (3)$$

Since $|w_{\lambda,1}(t)|$ is an even function, its Fourier transform can be written as

$$\Im\left(\left|w_{\lambda,1}(t)\right|\right) = \int_{-\infty}^{+\infty} \left|w_{\lambda,1}(t)\right| \cos(2\pi f_0 t)\,dt \qquad (4)$$

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Now since the function to be integrated is again even and the integral converges, we can write:

$$\Im(|w_{\lambda,1}(t)|) = \int_{-\infty}^{+\infty} |w_{\lambda,1}(t)| \cos(2\pi f_0 t) dt =$$

$$-\frac{2}{\sqrt{2\pi}} \int_0^{+\infty} -\frac{2t}{\lambda^2} e^{-t^2/\lambda^2} \cos(2\pi f_0 t) dt = \tag{5}$$

$$-\frac{2}{\sqrt{2\pi}} \int_0^{+\infty} \frac{d}{dt}\left(e^{-t^2/\lambda^2}\right) \cos(2\pi f_0 t) dt$$

The integral can be resolved analytically by parts:

$$W_{\lambda,1}^{a}(f) = \Im(|w_{\lambda,1}(t)|) =$$

$$-\frac{2}{\sqrt{2\pi}} \int_0^{+\infty} \frac{d}{dt}\left(e^{-t^2/\lambda^2}\right) \cos(2\pi f_0 t) dt =$$

$$-\frac{2}{\sqrt{2\pi}} \left[e^{-t^2/\lambda^2} \cos(2\pi f_0 t)\right]_0^{+\infty}$$

$$-\frac{4\pi f_0}{\sqrt{2\pi}} \int_0^{+\infty} e^{-t^2/\lambda^2} \sin(2\pi f_0 t) dt = \tag{6}$$

$$\frac{2}{\sqrt{2\pi}} - \frac{4\pi f_0}{\sqrt{2\pi}} \int_0^{+\infty} e^{-t^2/\lambda^2} \sin(2\pi f_0 t) dt =$$

$$\frac{2}{\sqrt{2\pi}} + \frac{2\pi f_0 \lambda}{\sqrt{2}} e^{-a^2} \Im(erf\ (ia))$$

$$a = -\pi\lambda f_0$$

where *i* is the imaginary unit. Finally, since the argument of the *erf* function is negative, the last part of the Eq. 6 can be approximated with the following series expansion:

$$W_{\lambda,1}^{a}(f) = \frac{2}{\sqrt{2\pi}} - \frac{2a^2}{\sqrt{2\pi}} e^{-a^2} \left(1 + \frac{a^2}{3} + \frac{a^4}{10} + \frac{a^6}{42} + \frac{a^8}{216} + \ldots\right) \tag{7}$$

The denominator terms of the approximation of the erf function are sequence A007680 in the On-line encyclopedia of integer sequences [10]. The expression of the Fourier transform of the rectified action potential has a zero that can be obtained analytically by imposing:

$$\left[W_{\lambda,1}^{a}(f_{DIP}^{R})\right]^2 = 0 \quad \Rightarrow \quad f_{DIP} \cong \frac{49}{53} \frac{1}{\pi\lambda} \tag{8}$$

Similarly, the -3 dB frequency is obtained as:

$$\left[W_{\lambda,1}^{a}(f_{-3dB}^{R})\right]^2 = \frac{1}{\pi} \quad \Rightarrow \quad f_{-3dB}^{R} \cong \frac{2}{5} \frac{1}{\pi\lambda} \tag{9}$$

In the same way, we obtain the frequency of the -3dB frequency for the unrectified action potential (high-pass filter):

$$\left[W_{\lambda,1}^{a}(f_{-3dB})\right]^2 = \frac{1}{\pi} \quad \Rightarrow \quad f_{-3dB} \cong \frac{17}{50} \frac{1}{\pi\lambda} \tag{10}$$

B. Simulations

The theoretical derivations were validated using a motor neuron and a surface EMG model.

The surface EMG model was based on a cylindrical volume conductor [11] and used in previous studies (e.g. [6, 12, 13]). The parameters of the surface EMG model were the same as described in [6]. Single muscle fibers action potentials were independently simulated and the surface-recorded, motor-unit potentials comprised the sum of the action potentials of the muscle fibers belonging to each motor unit. Surface EMG signals were computed at 4096 samples/s and had a bandwidth of approximately 10-500 Hz, similar to experimentally recorded signals. The signals were simulated in bipolar derivation with 10 mm inter-electrode distance. Changes in conduction velocity were simulated modifying the temporal support of the motor unit action potentials.

The motor neuron model was a modification of the one previous proposed by Fuglevand et al. 1993 [14]. However, the theoretical derivation previously reported is independent of the model used for the generation of the neural drive. It consists of a pool of 120 integrate-and-fire motor neurons that receive independent Gaussian synaptic inputs. The distribution of the recruitment thresholds was selected to be exponential [14]. The minimum and maximum discharge rates of the motor neurons were 8 and 35 pps, respectively. The level of normalized (with respect to maximum) excitation for which all motor neurons were recruited was 75 %. The force was simulated as a summation of twitch trains of each motor unit. The peak amplitude (P) and time-to-peak (T) were selected according to an exponential distribution [14]. In all simulations, the mean and the standard deviation of the synaptic input to all motor neurons corresponded to a generated force of approximately 5 % MVC and an average coefficient of variation (CoV) for the inter-spike interval (ISIs) of ~10 %. The resulting average CoV for the force signal was ~2 %.

Transfer functions of the raw and rectified motor unit action potentials were calculated using the fast Fourier transform with 4096 points. The power spectra of the raw and rectified surface EMG signals were calculated using Welch approach with Hanning windows of 1 s duration and 4096 points. For the rectified surface EMG, the mean value was removed before the calculation of the power spectrum. Cut off high pass (raw) and low pass frequencies were reported for the simulated motor unit action potentials. Frequencies of the first zero were also estimated and compared with the theoretical predictions.

III. Results

The theoretical derivation demonstrates that a rectified HR waveform contains zeros (dips) and attenuations that are dependent on the overall duration of the waveform.

Figure 1 provides an example of fitting for a simulated motor unit action potential using the first order HR waveform. In this example, the HR waveform with λ=0.007 (blue line) best fitted the simulated bipolar motor unit action potential (dashed grey line) shape. Fig 1A-B shows the fitting for the raw and the rectified potentials. The corresponding transfer functions are depicted in Fig 1C-D. In this case, it is evident a dip at approx. 40 Hz.

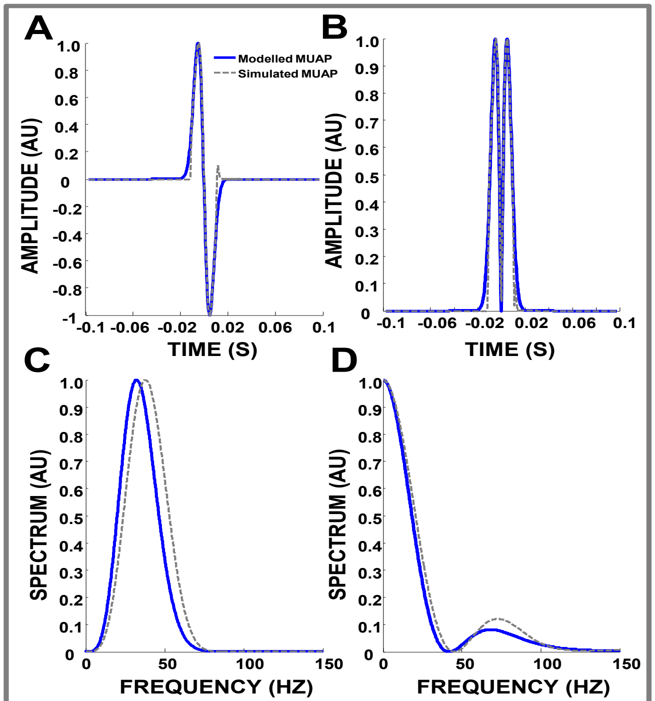

Fig. 1. Fitting of a simulated action potential with the first order HR function. A) Original waveforms. B) Rectified waveforms. C-D) Transfer function of the raw and rectified waveforms

Figure 2 provides an overview of the dependencies of the spectral parameters from the duration of the waveform.

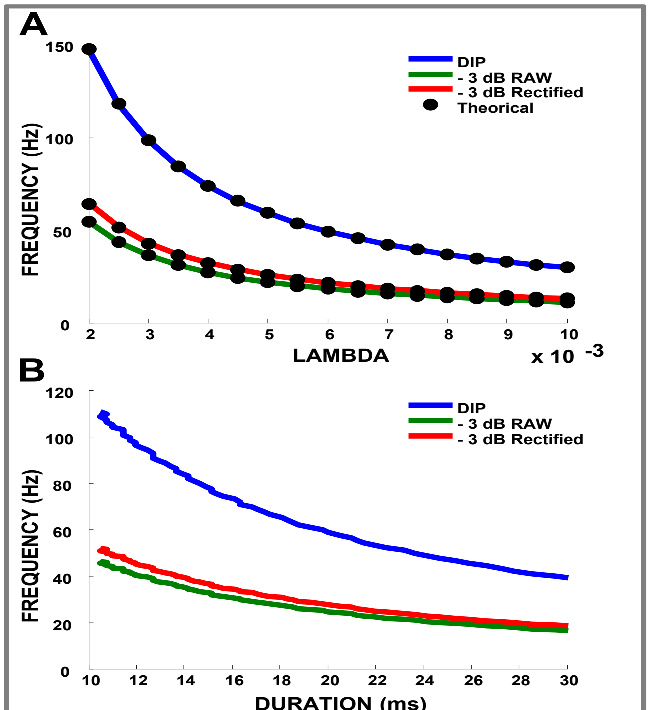

Fig. 2. Dip and attenuation (- 3 dB) frequencies for the theoretical (A) waveform. The duration of the waveform ranged between 10 and 40 ms. B) Same for the simulated waveform.

Fig 2A shows how the dip, the -3 dB low-pass frequency for the rectified HR waveform and the -3 dB high-pass frequency for the raw HR waveform are dependent on the parameter λ (that is related to the duration of the waveform). The colored lines report the values estimated directly from the transfer functions whereas the black points indicate the estimation computed with Eq. 8-10. Fig 2B shows the same results for a simulated motor unit action potential with variable conduction velocity. Interestingly, the low and high pass frequencies for the rectified and raw potentials are very similar.

The previous results demonstrated that the rectified bipolar motor unit action potential has a low-pass behavior with a dip frequency that depends on the duration of the waveform. According to Eq. 2, the spectral properties of the rectified motor unit action potential shapes can influence the spectral content of the rectified EMG signal. To test this effect, we simulated the surface EMG signal with the motor neuron and EMG computational model.

Figure 3 shows an example of simulation. Fig 3A shows the average discharge rates of the active motor neurons (8-13 pps). Fig 3B-C show the generated force (~ 5 % MVC) and the resulting surface EMG signal.

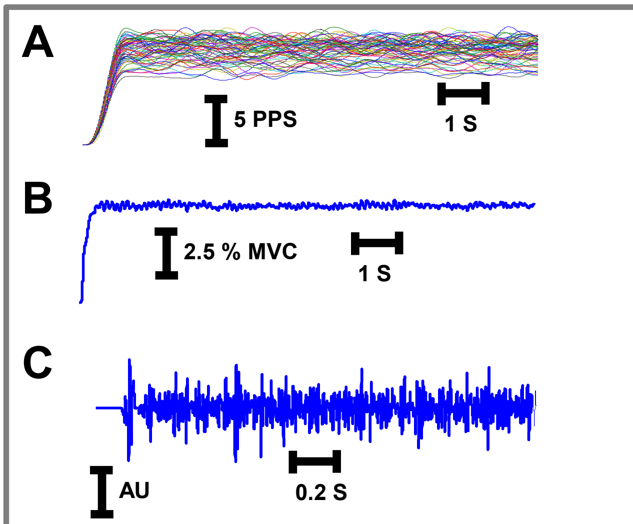

Fig. 3. Example of simulated data. A) Average discharge rates of the active motor neurons. B) Net output force. C) Raw EMG signal.

A change of the conduction velocity and duration of the motor unit action potentials, as the one that can occur during muscle fatigue, can heavily influence the spectral properties of the rectified surface EMG signal. Figure 4 shows an example of the power spectra of rectified EMG signals with different conduction velocity. Fig 4A-C clearly show how the spectral content, as -3 dB bandwidth and dip frequencies, can shift with a change of the global muscle fiber conduction velocity.

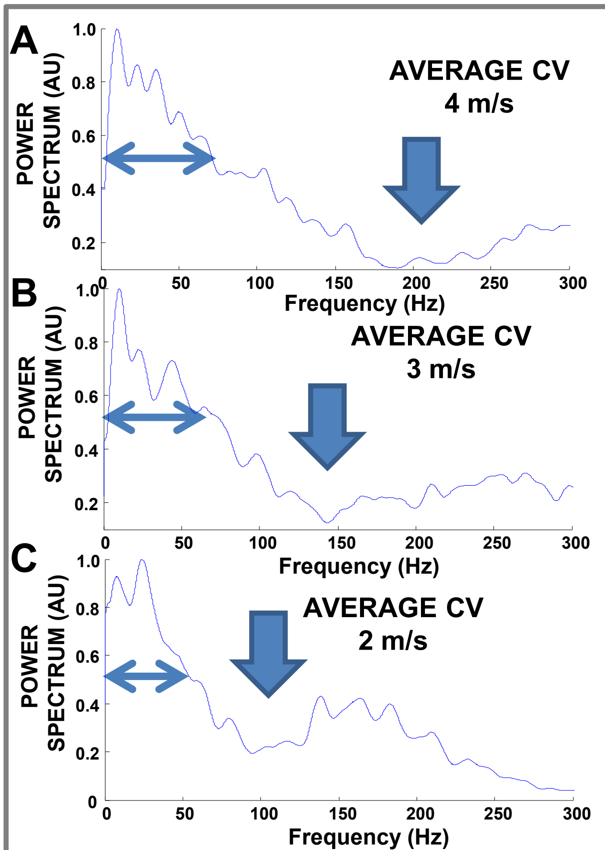

Fig. 4. Power spectrum of rectified EMG signals with different average muscle fiber conduction velocity.

IV. Discussion

Rectification is a common procedure that is used to extract the neural oscillations contained in the EMG signals. In this study, we demonstrated how the spectral properties of the rectified EMG depend on those of the rectified motor unit action potentials. Changes in the duration of the action potentials, such as those observable at fatigue, can heavily affect the spectral content of the rectified EMG signal.

This problem may have a minor impact for low frequencies (close to DC), but can cause a filtering of higher frequencies (alpha and beta bands) that depends on the motor unit action potential duration. These results are particularly relevant for coherence analysis (EEG-EMG and EMG-EMG) since they indicate that the magnitude of EMG coherence after rectification cannot be compared across conditions that have different motor unit action potential durations (conduction velocities).

V. Conclusion

In this study, we demonstrated that the motor unit action potential has an important influence on the rectified EMG signal. Therefore, in order to properly compare the spectral content of the rectified EMG signals across conditions, the knowledge of the properties of the motor unit action potentials is necessary.

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