Signed-Gradient Adaptive Step Size LMS Algorithm for Biomedical Applications

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***Abstract*—Gradient adaptive step size adaptive filters have been widely used to adapt different biomedical application environments and obtain useful life signals from serious ambient noise and interferences. In order to further improve the signal-to-noise ratio (SNR) of the life signals, this paper presents a class of signed-gradient adaptive step size least mean square (LMS) adaptive filters. The proposed algorithms introduce a sign function to replace the gradient of squared error in the step size updating process of the gradient adaptive step size LMS adaptive filters. The performance of both gradient and signed-gradient algorithms with dual adaptive filters is compared by extracting heartbeat signals from ambient noise in stethoscopes. Simulation results demonstrate that though the signed-gradient adaptive step size LMS algorithm converges at a slower rate at the early stage of iteration, it has a smaller mean squared error (MSE) at the stage of convergence, thus achieves a higher SNR.**

I. Introduction

Good adaptation and effective noise reduction are important criteria of an adaptive filtering system to remove ambient noise or extract life signals from serious noises and interferences [1-7]. Due to the diversity of the application environments, a least mean square (LMS) adaptive filter with a fixed step size always suffers from slow convergence and large error, when extracting useful signals from noises or interferences. Therefore, an adaptive filter is better able to adjust its step size on the basis of different application environments. The step size should be large at the early stage of iteration for fast convergence and becomes small at the stage of convergence to obtain useful signals with high signal-to-noise ratio (SNR).

Gradient adaptive step size adaptive filters have been proposed to achieve good adaptation, which the step-size parameter is adjusted automatically by using gradient descent technique [8-13]. Benveniste et al first proposed gradient adaptive step size algorithm for LMS adaptive filter [8]. The algorithm is capable of calculating the exact gradient, resulting in good adaptation. Then Mathews and Xie proposed a new gradient adaptive step size LMS algorithm with lower computational complexity [10]. However, due to the rough estimation of gradient, its performance is not as good as Benveniste's algorithm. Ang and Farhang-Boroujeny also proposed a new method of estimating the gradient in order to simplify the equations of Benveniste's algorithm [12]. The accuracy of gradient estimation for the method is better than that of Mathews' algorithm, but worse than that of Benveniste's algorithm. Thus its performance is between those of Mathews' and Benveniste's algorithms. Previously, we proposed a gradient adaptive step size algorithm with dual LMS adaptive filters. The algorithm is different from the traditional methods using gradient descent technique in that the gradient is measured with two LMS adaptive filters [14]. Nevertheless, it still has a low computational complexity. Simulation results demonstrated that this algorithm achieves as good adaptation as Benveniste's algorithm.

As for adaptive filtering systems in biomedical applications like extracting life signals from serious noises and interferences, good adaptation at the early stage of iteration does not necessarily lead to small error at the stage of convergence. The reason is that a steady state after convergence is easily destroyed when sudden large signals such as repeated heartbeats and QRS complexes of ECG appear [6]. In this paper, a class of signed-gradient adaptive step size LMS adaptive filters are proposed. Unlike the traditional sign LMS adaptive filters [15-19], the proposed algorithms introduce sign function to replace the gradient of squared error in the updating process of step size. In this paper, the performance of adaptation and noise reduction of gradient and signed-gradient adaptive step size LMS algorithms with dual adaptive filters are compared.

The paper is organized as follows. LMS algorithm and several gradient adaptive step size LMS algorithms are described in Section 2. Details of the new class of signed-gradient adaptive step size LMS algorithms are given in Section 3. Section 4 presents the comparison between the simulation results of gradient and signed-gradient adaptive step size LMS algorithms with dual adaptive filters. The conclusion is made in the last section.

II. LMS and Gradient Step Size LMS Algorithm

A. LMS algorithm

The block diagram of an adaptive filter is illustrated in Fig. 1 [20]. The filter is a finite impulse response (FIR) filter with length N. The vector of tap inputs at time n is denoted by $X(n)$, which includes the tap inputs $x(n)$, $x(n\text{-}1)$,…, $x(n$-

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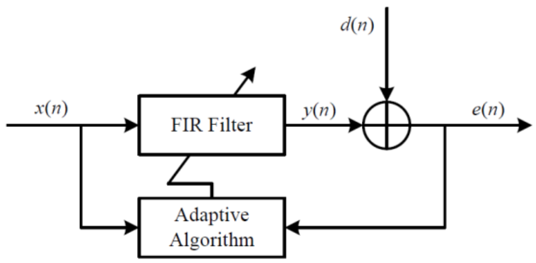

Figure 1. Block diagram of an adaptive filter

N+1). The weight vector at time n is denoted by $W(n)$ which includes tap weights $w_0(n)$, $w_1(n)$,…, $w_{N-1}(n)$. $x(n)$ is the reference input, $d(n)$ is the desired response, and $y(n)$ is the corresponding estimate of $d(n)$ at the filter output. By comparing the desired response and its estimate, an estimation error can be obtained:

$$\begin{aligned} e(n) &= d(n) - y(n) \\ &= d(n) - W^T(n)X(n) \end{aligned} \tag{1}$$

where $(\cdot)^T$ is the vector transpose operator.

The LMS algorithm is described by the equation [20]:

$$W(n+1) = W(n) + \mu(n)e(n)X(n) \tag{2}$$

where $\mu(n)$ is the step size of the LMS adaptive filter, which controls the convergence rate. For traditional LMS adaptive filter, $\mu(n)$ is a fixed value. The condition of the algorithm achieving convergence is $0 < \mu(n) < 1/\lambda_{max}$, where λ_{max} is the maximal eigenvalue of the autocorrelation matrix of $X(n)$.

B. Gradient adaptive step size LMS algorithms

Gradient adaptive step size LMS adaptive filter adapts the step size sequence using a gradient descent algorithm so as to reduce the squared-estimation error at each iteration. The update equation for step size $\mu(n)$ is given as [8, 10, 12]

$$\mu(n) = \mu(n-1) - \frac{\rho}{2}\nabla_\mu e^2(n) \tag{3}$$

where ρ is a small positive constant that controls the adaptive behavior of step size, $\nabla_\mu e^2(n)$ is the gradient of squared error at time n. Tab. I gives the equations of gradient for Mathews', Benveniste's and Ang's gradient adaptive step size LMS algorithms [8, 10, 12], and the gradient adaptive step size LMS algorithm with dual adaptive filters (or gradient dual-filter algorithm) [14]. In the table, I is an identity matrix; a is a constant smaller than but close to one; $\tilde{\nabla}_\mu e^2(n)$ is the approximation of gradient for the gradient dual-filter algorithm; the suffixes "w" denote the work filter and "r" denote the reference filter; and $\Delta\mu$ is the step size difference of the two filters.

III. Signed-Gradient Adaptive Step Size LMS Algorithm

The updated equation of step size of the new class of signed-gradient adaptive step size LMS algorithms is given by

$$\begin{aligned} \mu(n) &= \mu(n-1) - \frac{\rho}{2}\operatorname{sgn}(\nabla_\mu e^2(n)) \\ &= \mu(n-1) - \rho' \operatorname{sgn}(\nabla_\mu e^2(n)) \end{aligned} \tag{4}$$

where ρ' is the step size increment which is a positive constant, and

$$\operatorname{sgn}(a) = \begin{cases} 1, & if\ a > 0 \\ 0, & if\ a = 0 \\ -1, & if\ a < 0. \end{cases} \tag{5}$$

As for the signed-gradient adaptive step size LMS algorithm with dual adaptive filters (or signed-gradient dual-filter algorithm), its updated step size is described by

$$\mu_w(n) = \mu_w(n-1) - \rho' \operatorname{sgn}(\tilde{\nabla}_\mu e^2(n)). \tag{6}$$

If $\Delta\mu$ is a positive value, the equation above becomes

$$\mu_w(n) = \begin{cases} \mu_w(n-1) - \rho', & if\ |e_w(n)| > |e_r(n)| \\ \mu_w(n-1), & if\ |e_w(n)| = |e_r(n)| \\ \mu_w(n-1) + \rho', & if\ |e_w(n)| < |e_r(n)| \end{cases} \tag{7}$$

where |•| is an absolute symbol.

IV. Simulations

In this section, we use signed-gradient dual-filter algorithm as an example to show the performance of the signed-gradient adaptive step size LMS algorithms.

We use the heartbeat signal extraction from ambient noise of electronic stethoscopes to compare the performance of gradient and signed-gradient adaptive step size LMS algorithms. In an active noise cancellation (ANC) system using adaptive filtering, the desired response $d(n)$ of the adaptive filter is the combination of the biomedical signals $s(n)$, and a noise derived from the reference input $x(n)$, which is the ambient noise, after passing an unknown system. Here we consider the unknown system as a five-point FIR filter

TABLE I. The Equations of $\varphi(n)$ for Different Gradient Algorithms

Algorithm	*Equations*
Mathews' [10]	$\nabla_\mu e^2(n) = -2e(n)X^T(n)\varphi(n)$ $\varphi(n) = e(n-1)X(n-1)$
Benveniste's [8]	$\nabla_\mu e^2(n) = -2e(n)X^T(n)\varphi(n)$ $\varphi(n) = [I - \mu(n-1)X(n-1)X^T(n-1)] \cdot \varphi(n-1) + e(n-1)X(n-1)$
Ang's [12]	$\nabla_\mu e^2(n) = -2e(n)X^T(n)\varphi(n)$ $\varphi(n) = a\varphi(n-1) + e(n-1)X(n-1)$
Gradient dual-filter [14]	$\tilde{\nabla}_\mu e^2(n) = \dfrac{e_w^2(n) - e_r^2(n)}{\Delta\mu}$

with coefficients [10-11]:

$$W_o = \{0.1, \quad 0.3, \quad 0.5, \quad 0.3 \quad 0.1\}. \qquad (8)$$

The input signal $x(n)$ is zero-mean, white, and Gaussian noises with a variance of 0.3. The desired response signal $d(n)$ is obtained by adding the output of the system in (8) with the heartbeat signal extracted from 3M Littmann stethoscope [21]. The heartbeat signal with the sampling rate of 11,025 Hz is shown in Fig. 2a.

The parameters used are M=10^3, N=5, $\Delta\mu$=10^{-6}, $\mu(0)$= 10^{-3}, and ρ =0.0002 for the gradient dual-filter algorithm, and ρ' =0.0001 for the signed-gradient dual-filter algorithm. One hundred of independent runs and 40,000 samples per run are used in the simulation. All results are the averages after 100 runs.

Fig. 2b shows the valid gradient comparison between the gradient and signed-gradient dual-filter algorithms. The valid gradient of gradient dual-filter algorithm is its gradient function $\tilde{\nabla}_\mu e^2(n)$. As for the signed-gradient dual-filter algorithm, the valid gradient is its sign gradient function $\mathrm{sgn}(\tilde{\nabla}_\mu e^2(n))$. It is shown in Fig. 2b that the gradient function of the gradient dual-filter algorithm changes more rapidly when the heartbeat signals are present while that of the signed-gradient dual-filter algorithm stays within the range of -1 to 1.

Fig. 2c shows the mean behavior of the step size of gradient and signed-gradient dual-filter algorithms. As shown in the figure, the step size of gradient dual-filter algorithm increases more rapidly than the signed-gradient dual-filter algorithm at the early stage of iteration, which indicates that a faster convergence is achieved. Although the signed-gradient dual-filter algorithm converges slower, its step size decreases more quickly than the gradient dual-filter algorithm and achieves a smaller step size when the heartbeat signals are present. Small step size is more desirable as it would result in smaller error and lower signal distortion. Interestingly, after the heartbeat signal is gone, the step size of gradient dual-filter algorithm remains constant, but it is not the case for signed-gradient dual-filter algorithm. After the first heartbeat signal, its step size keeps decreasing. And after the second and the subsequent heartbeat signals, the step size first increases rapidly and then decreases again.

In order to better compare the noise reduction performance of the two algorithms, we calculated the mean squared error (MSE) and SNR by

$$MSE = \frac{1}{L}\sum_{l=0}^{L-1}\left(e_l(n) - s_l(n)\right)^2 \qquad (9)$$

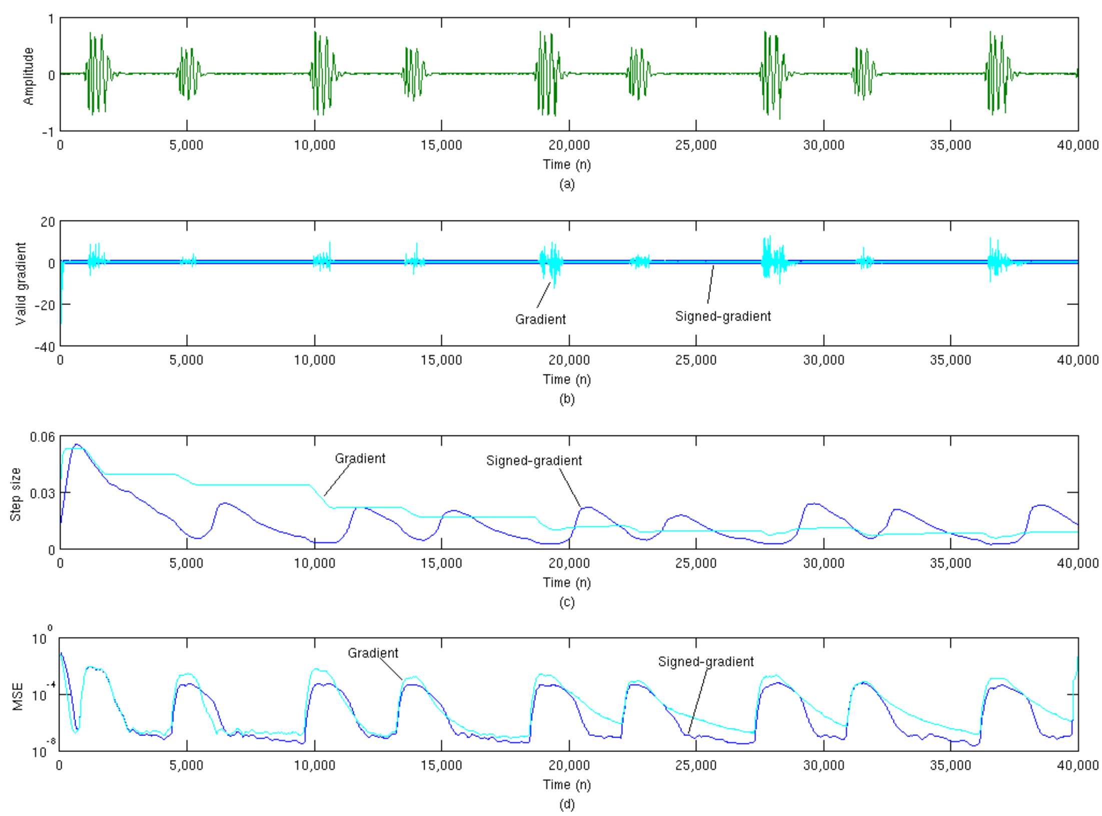

Figure 2. Comparison of the simulated results of gradient and signed-gradient dual-filter algorithms when ρ = 0.0002 and ρ' = 0.0001: (a) heartbeat signal from stethoscope, (b) valid gradient for one run, (c) mean step size behavior after 100 runs and by 200 points averaging, (d) Mean squared error after 100 runs and by 200 points averaging

$$SNR = 10\log_{10}\left(\frac{1}{L}\sum_{l=0}^{L-1}\left(\frac{\text{var}(s_l(n))}{\text{var}(e_l(n)-s_l(n))}\right)\right) \quad (10)$$

where l and L denote the number and total number of the independent runs respectively, var is the function of variance. In the case, L is 100. The results of MSE are shown in Fig. 2d. The figure clearly shows that the signed-gradient dual-filter algorithm achieves a smaller MSE, no matter the heartbeat signals are present or not. Tab. II shows the SNR comparison between gradient and signed-gradient dual-filter algorithms during the time period from $1\text{x}10^4$ to $4\text{x}10^4$. The signed-gradient dual-filter algorithm achieves a higher SNR of more than 5 dB.

TABLE II. SNR COMPARISON OF GRADIENT AND SIGNED-GRADIENT DUAL-FILTER ALGORITHMS DURING TIME PERIOD FROM $1\text{x}10^4$ TO $4\text{x}10^4$

Algorithm	***SNR(dB)***
Original signals	-6.95
Gradient, ρ=0.0002	19.89
Signed-gradient, ρ' =0.0001	25.52

V. CONCLUSION

This paper presents a new class of signed-gradient adaptive step size LMS algorithm, which uses the sign of gradient to update step size. Traditional gradient adaptive step size LMS algorithms can be easily transformed to the corresponding signed-gradient algorithms by replacing the gradient with the signed gradient. Gradient and signed-gradient dual-filter algorithms are simulated to compare the difference of adaptation and noise reduction performance in extracting heartbeat signals from ambient noise in stethoscope application. Simulation results demonstrate that compared with gradient dual-filter algorithm, signed-gradient dual-filter algorithm converges with a slower rate at the early stage of iteration, but achieves a better overall performance of noise reduction at the stage of convergence with a higher SNR.

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