Low-dose Computed Tomography Image Denoising based on Joint Wavelet and Sparse representation

Samira Ghadrdan, Javad Alirezaie*- *Senior Member IEEE,* Jean-Louis Dillenseger, *and* Paul Babyn

Abstract— **Image denoising and signal enhancement are the most challenging issues in low dose computed tomography (CT) imaging. Sparse representational methods have shown initial promise for these applications. In this work we present a wavelet based sparse representation denoising technique utilizing dictionary learning and clustering. By using wavelets we extract the most suitable features in the images to obtain accurate dictionary atoms for the denoising algorithm. To achieve improved results we also lower the number of clusters which reduces computational complexity. In addition, a single image noise level estimation is developed to update the cluster centers in higher PSNRs. Our results along with the computational efficiency of the proposed algorithm clearly demonstrates the improvement of the proposed algorithm over other clustering based sparse representation (CSR) and K-SVD methods.**

I. Introduction

One of the major recent goals in CT research and image processing is to lower the need for harmful patient radiation dose by reducing the noise on the low- dose CT images by post processing. Due to a complex relation between image noise and scan parameters and spatial position [1], finding a distribution of noise in the final CT image is difficult. The noise is usually unknown and noise variance is a variable parameter. Different algorithms have been proposed to reduce the CT noise. One type removes noise in the projection data before image reconstruction while in the second category, algorithms reduce noise during the CT reconstruction phase. These algorithms perform denoising through optimizing statistical objective functions [2, 3]. The most common methodology is noise reduction algorithms of the reconstructed CT images. A critical aspect is to preserve edges and small important structures for diagnosis while denoising. The conventional edge-preserving methods in frequency domain are wavelet-based methods and in spatial domain are partial differential equation (PDE) based methods [4, 5].

Sparse representation has been widely used as a dominant tool for image noise removal allowing the preservation of important information and edges. This method is a non-local model which reconstructs the signal based on a set of basic vectors called dictionary atoms. A suitable dictionary can be selected by utilizing either analytical or adaptive dictionary techniques. Adaptive dictionaries are constructed based on training of different patches of the noisy image. In contrast, analytical dictionaries are fixed with regards to the nature of the image using stationary basis functions. The well-known adaptive dictionary called K-SVD method [6] proposed by Elad and Aharon is the state of the art in this field. On the other hand, Non-Local models such as Non Local Means (NLM) [7] making use of the repetitive structures in an image and by exploring the similarity between patches have led to a successful denoising algorithm which among them BM3D [8] has shown the superior results. Combining these two complementary models, a clustering based sparse representation (CSR) algorithm has been proposed [9]. CSR algorithm unifies both models and formulates a double header l_1-optimization problem. Key advantage of this proposed method includes both sparsity and clustering (location related constraint) thus generating a sparser solution and better denoising results.

In this paper we propose an approach to combine conventional methods and sparse representation to denoise low dose CT images more efficiently. Using wavelets, we extract the features that are most suitable for denoising and edges preservation. The dictionary atoms are learned from k-means clustering and Principal Component Analysis (PCA) of each cluster. In this process we can find the accurate clusters and construct the dictionary with fewer atoms. A single image noise level estimation is developed in the algorithm to update the cluster centers more accurately. This method performs effectively for removing additive Gaussian noise from images, and has also been adapted to the non-Gaussian noise in CT images.

The remainder of the paper is organized as follows. Section II reviews the sparse representation and clustering-based sparse representation (CSR) denoising technique. Section III contains the algorithmic description of the proposed method. Section IV contains the main results of the paper on both synthetic and medical CT images along with a simulation-based study of its performance. Concluding remarks are given in Section V.

II. Related Works

A. Sparse Representation

Utilizing sparse representation framework is one of the state of the art techniques for image denoising. We can express an image by a sparse linear combination of dictionary atoms which are derived from the noisy image itself.

$$\boldsymbol{y} = \boldsymbol{D\alpha} \tag{1}$$

Samira Ghadrdan and Javad Alirezaie* are with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, ON, M5B2K3, Canada (e-mail:samira.ghadrdan@ryerson.ca; javad@ryerson.ca, phone: 416-979-5000; fax: 416- 979-5280).

Paul Babyn is with the Department of Medical Imaging, University of Saskatoon Health Region, Royal University Hospital. Saskatoon, SK, S7N 0W8 Canada (e-mail: paul.babyn@saskatoonhealthregion.ca).

Jean-Louis Dillenseger is with the Laboratoire Traitement du Signal et de l'Image, Université de Rennes 1, 35042 Rennes, France (e-mail : jean-louis.dillenseger@univ-rennes1.fr).

(Asterisk indicates corresponding author)

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where $\boldsymbol{y}$ is the vectorized noisy image contaminated by an additive white Gaussian noise and $\boldsymbol{D}$ is constructed from the training patches of the noisy image and α is the sparse vector. The sparsest solution to the above problem can be found by the following optimization problem:

$$\widehat{\boldsymbol{\alpha}} = argmin \parallel \boldsymbol{\alpha} \parallel_0 \ subject\ to\ \ \boldsymbol{D\alpha} = \boldsymbol{y} \tag{2}$$

where $\parallel\parallel_0$ denotes the l_0-norm and shows the number of non-zero elements in a vector.

we can change the formulation to the following variational problem for the purpose of image denoising.

$$\hat{\alpha} = \mathrm{argmin}_{\alpha} \parallel \boldsymbol{y} - \boldsymbol{D\alpha} \parallel_2^2 + \lambda \parallel \alpha \parallel_1 \tag{3}$$

where λ is the standard lagrangian multiplier. Many attempts to solve the NP-Hard optimization problem and design the proper dictionary are reported in literature [6].

B. Clustering-based Sparse Representation

Due to the nature of sparse representation, the sparse coefficients are not randomly distributed. This is likely where the idea came to Dong et al. [9] to combine sparse representation with location related methods? to obtain a higher order of sparsity. They proposed to unify sparsity and clustering and formulated a double header l_1-optimization problem:

$$\hat{\alpha}, \hat{\beta} = \mathrm{argmin}_{\alpha,\beta_k} \parallel \boldsymbol{y} - \boldsymbol{D\alpha} \parallel_2^2 + \lambda_1 \parallel \alpha \parallel_1 + \lambda_2 \textstyle\sum_{k=1} \sum_{i \in C_k} \parallel \alpha_i - \beta_k \parallel_1 \tag{4}$$

where β_k is used to represent the center of each cluster $\mu_k$with respect to the same dictionary as α. The solution to the above problem can be reached by an iterative shrinkage algorithm [10].

III. Methodology

In low-dose CT images, the structural details and information is still available, but the noise often blurs the edges or confuses some of the detailed characteristics. Sparse representation and an adaptive dictionary can be utilized to remove the noises effectively. To improve the performance of the sparse representation we want to use a preprocessing method to magnify important structures and details of images in order to learn a more accurate adaptive dictionary. Inspiring from the repeated patterns especially at the edges of medical images, structural clustering with sparse representation is combined. In this case the denoising algorithm can benefit from grouping the similar patches and present higher sparsity in denoising.

A. Wavelet Preprocessing

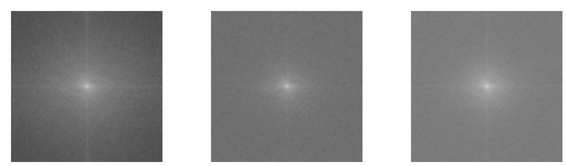
One of the most used conventional method to denoise the medical images is wavelet denoising since it preserves edges and important image structures. Wavelet denoising is usually done by thresholding the wavelet coefficients, thus, it is really important how to choose the coefficients to serve the best for edge preserving while keeping the frequencies that the algorithm is working based on them. The global threshold can be calculated by

$$\lambda = \sigma\sqrt{2logN} \tag{5}$$

where σ is the noise vaiance and N the size of image. The global threshold is not really of interest since the result is an over-smoothed image. To overcome the smoothing problem and finding the optimal threshold, the *square root balance-sparsity norm* approach is used. First, a thresholds array t, which contains uniformly distributed values between 0 and 1 is defined. The t array is used to define two curves: the percentage of L2-norm recovery (the measure of the energy loss after the denoising process using the value in t) and the percentage of the relative sparsity (the number of resulting zero coefficients in the denoised image). The intersection of two curves is t_{opt} and *the square root balance-sparsity norm* threshold is defined using equation (6),

$$\lambda = \sqrt{t_{opt}} \tag{6}$$

By introducing this balance to the system the wavelet denoising preserves the detailed information and at the same time reduces the unwanted noise which makes the clustering step difficult and less accurate. Fourier spectrum of the noisy image (Man) and clustering step (dictionary leaning) input in CSR and our proposed method, joint Wavelet and CSR (WCSR), are compared in Figure 1. The figure illustrates the preservation of the high frequencies which are the selected features for clustering and dictionary learning.



(a) (b) (c)

Figure 1. Fourier spectrum of the a) noisy image (Man), b) clustering based sparse representation (CSR) and c) proposed joint wavelet and clustering sparse representation (WCSR)

As it is shown in the image high frequencies are retained better than CSR and the algorithm preserves the important information and edges at the same time.

B. Single Image Noise Level Estimation

In the iterative solution to the double header l_1-optimization problem using surrogate function [9] there are two regularization parameters τ_1, τ_2 defined which are inversely proportional to signal-to-noise –ratio (SNR).

$$\tau_1 = c_1 \frac{\sigma_\omega^2}{\sigma_\alpha}, \tau_2 = c_2 \frac{\sigma_\omega^2}{\sigma_\gamma} \tag{7}$$

where σ_ω^2 is the noise variance and $\gamma = \alpha - \beta$ and c_1, c_2 are constants. The algorithm uses these regularization parameters to update the cluster centers so it is really important to have a precise estimation of the noise variance. We have deployed a new algorithm to improve the noise variance estimation based on the sole noisy image [11]. Utilizing a patch based noise level estimation, the weak patches from a single noisy image are selected. The selection is based on the gradients of the patches and their statistics and then PCA is used to do the noise estimation from the selected patches. It is a fast and accurate method to estimate the variance of the noise in the lower variances (less than $\sigma = 10$) and improves the performance of estimation and denoising algorithm.

C. Proposed Algorithm

In order to extract important information and preserve edges which both are significant details for diagnosis in CT images, wavelet denoising as a preprocessor and sparse representation based on dictionary learning and clustering is proposed. As the results show in Figure 1. the proposed algorithm has maintained and improved the high frequency information and while preserving and improving the detailed information and edges as well. Improved detailed information of the image helps to learn an adaptive dictionary with a reduced number of atoms, so fewer numbers of clusters are needed. This reduces the computational complexity of the algorithm as well. The algorithm will have the following structure with two dependent loops. In the first step we have to learn the dictionary which will be done by wavelet analysis and clustering and we repeat that for k times. The next step will be using the designed dictionary to denoise the image which will be done for l times. The initialization contains the definition of number of clusters with significant effect on the denoising process. The noise level estimation will be done in the second step each time to help the accurate update of τ_1, τ_2. Proposed method flowchart is shown in Figure 2.

Pseudo-Algorithm of the proposed method:

Initialize: $\boldsymbol{X}_{den} = \boldsymbol{y}$ **,** Number of clusters;

Set $k = 1$

1. **Wavelet and Clustering;**
   - Wavelet feature extraction and denoising;
   - Feature extraction using HPF;
   - K-means clustering;
   - Dictionary learning through PCA;

2. **Sparse representation and noise estimation**
   - Set $l = 1$, Begin Iterative regularization;
   - Single image noise σ_ω^2 estimation;
   - Sparse representation and estimation of τ_1, τ_2 through equation (7);
   - Centroid estimate update β_k;
   - Denoised Image update, $\boldsymbol{X}_{den}$;
   - Set $l = l + 1$;

Set $k = k + 1$;

IV. RESULTS

The proposed image denoising method is tested on Lena ,Couple and Man in Table 1. with different amount of noise variance. In all the experiments, the dictionary size for K-SVD algorithm is 64; CSR is tested with 30 and 64 dictionary atoms and WCSR method with just 30. It compares the K-SVD method [6] with CSR [9] and the proposed denoising method, WCSR. It can be observed that the proposed denoising method achieves better performance in terms of PSNR and less than half of required clusters.

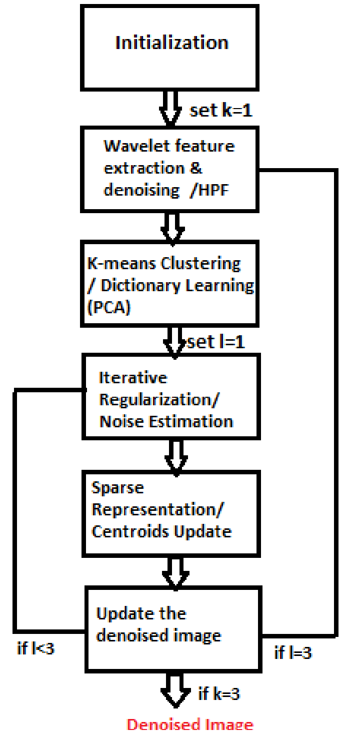


Figure 2. Flowchart of the proposed method

Figure 3. Top left: Original Image, Top right: Noisy image(PSNR=22.11), Bottom left: Denoised image with CSR(PSNR=30.56, SSIM=0.8306) and Bottom right: Denoised image with WCSR (PSNR=30.67, SSIM=0.8361)

In respect to Structural Similarity (SSIM) Figure 3. shows the improvement over CSR which implies better perceptual quality of the denoised image. Figure 3 shows that the denoised images using CSR and the proposed methods. It can be seen that the proposed denoising scheme preserves the structures better and therefore has better perceptual image quality. The proposed algorithm is tested on medical images as well shown in Figure 4. This contains the results of the denoising on a CT image with the additive white Gaussian noise with noise variance σ=20 and as it is shown the PSNR improved from 22.11 dB to 32.84dB compared to K-SVD with PSNR 31.61dB. The result of the denoising algorithm over low-dose CT image is shown in Figure 5.

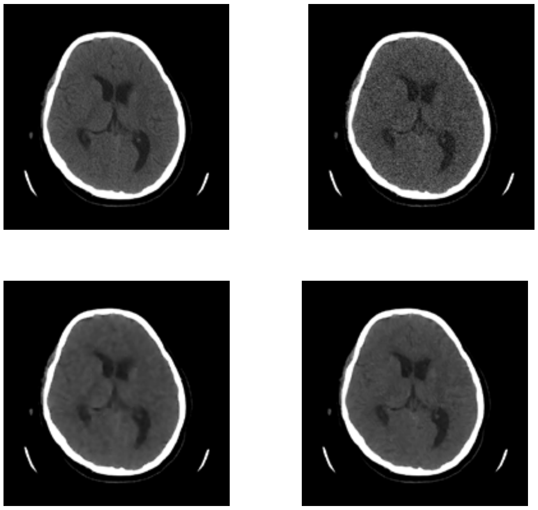


Figure 4. Top left: Original Image, Top right: Noisy image (PSNR = 22.11), Bottom left: Denoised image with K-SVD (PSNR = 31.61) and Bottom right: Denoised image with WCSR (PSNR = 32.84).

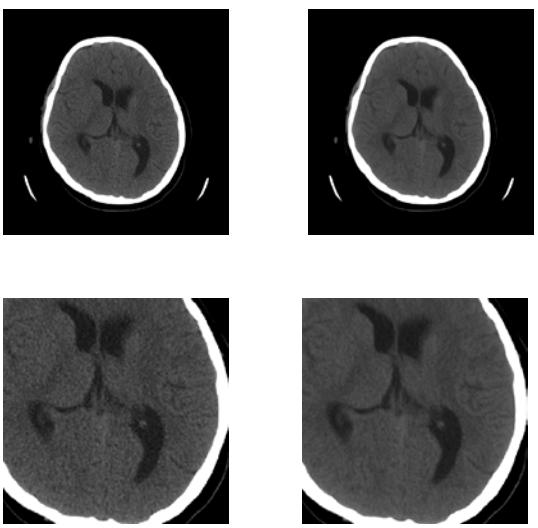


Figure 5. Top left: Low-Dose CT image and Bottom Left: Zoomed image, Top right: Denoised image with WCSR and Bottom right: Zoomed image

As we expected the results all confirm that the denoising algorithm works well for CT images and improves the quality of the image both with respect to PSNR and visually for diagnosis.

V. Conclusion

In this paper, we presented a joint wavelet sparse representation denoising algorithm on the low-dose CT images. In the first step utilizing wavelets the algorithm surpasses the noise successfully while maintaining the important information and preserving the edges. This step prepares the image for clustering and dictionary learning. As a result of having higher quality image with lower noise we can learn a better dictionary with fewer number of clusters which brings higher computational efficiency. Deploying a new noise level estimation to the algorithm improves the performance for lower noise variances and makes it suitable for low-dose CT images. The experimental results show that the performance of our algorithm is better than CSR algorithm in terms of both PSNR and SSIM with lower computational complexity. It also confirms that our denoising algorithm works well for CT images, improving the quality of the image both with respect to PSNR and visually for diagnosis. The medical images are under review for ROC analysis; results will be presented in future reports.

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TABLE I. After The PSNR (dB) results for different denoising methods. In each cell, the results of four denoising algorithms are reported. Top left: CSR(64 clusters)[9]; Top right: CSR(30 clusters)[9]; Bottom left: K-SVD [6]; Bottom right: WCSR

σ/PSNR	**5/34.15**		**10/28.13**		**15/24.61**		**20/22.11**		**25/20.17**		**30/18.59**	
Lena512	38.74	38.75	35.90	35.88	34.20	34.19	32.96	32.94	31.98	31.96	31.16	31.14
	38.60	**38.82**	35.47	**35.96**	33.70	**34.28**	32.27	**33.06**	31.20	**32.07**	30.46	**31.23**
Couple	37.41	37.40	33.95	33.93	32.00	31.94	30.60	30.59	29.52	29.51	28.62	28.63
	37.30	**37.46**	33.48	**33.95**	31.44	**32.03**	29.96	**30.67**	28.93	**29.62**	28.07	**28.74**
Man	37.78	37.75	33.96	33.94	31.91	31.81	30.56	30.53	29.56	29.53	28.75	28.76
	37.50	**37.85**	33.55	**33.98**	31.44	**32.02**	30.05	**30.67**	29.02	**29.69**	28.23	**28.89**