Hybrid Frequency and Phase Coding for a High-Speed SSVEP-Based BCI Speller

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***Abstract*— Steady-state visual evoked potential (SSVEP)-based brain-computer interfaces (BCIs) have potential to realize high-speed communication between the human brain and the external environment. Recently, multiple access (MA) methods in telecommunications have been introduced into the system design of BCIs and showed their potential in improving BCI performance. This study investigated the feasibility of hybrid frequency and phase coding methods in multi-target SSVEP-based BCIs. Specifically, this study compared two hybrid target-coding strategies: (1) mixed frequency and phase coding, and (2) joint frequency and phase coding. In a simulated online BCI experiment using a 40-target BCI speller, BCI performance for both coding approaches were tested with a group of six subjects. At a spelling speed of 40 characters per minute (1.5 seconds per character), both approaches obtained high information transfer rates (ITR) (mixed coding: 172.37±28.67 bits/min, joint coding: 170.94±28.32 bits/min) across subjects. There was no statistically significant difference between the two approaches (p>0.05). These results suggest that the hybrid frequency and phase coding methods are highly efficient for multi-target coding in SSVEP BCIs with a large number of classes, providing a practical solution to implement a high-speed BCI speller.**

I. INTRODUCTION

Recently, the steady-state visual evoked potential (SSVEP)-based brain-computer interface (BCI) has attracted much attention for its advantages such as little user training, ease of use, and high information transfer rate (ITR) [1, 2]. In SSVEP-based BCIs, users are asked to fixate on one of multiple visual flickers tagged with different stimulation properties (e.g., frequency), and a gazed target can be identified through analyzing the SSVEPs elicited by the target stimulus. Currently, frequency coding and phase coding are the two most popular approaches to implement multi-target coding with SSVEPs. In BCIs using frequency coding, all stimuli flicker simultaneously at different frequencies [3-5]. In BCIs using phase coding, visual stimuli typically comprise multiple flickers at the same frequency but with different initial phases [6-8].

The stimulus coding method plays an important role in SSVEP-based BCIs [1, 2]. Recently, multiple access (MA) methods in telecommunications have been introduced into the system design of BCIs and showed their potential in improving BCI performance [9]. Typical MA methods such as time division multiple access (TDMA), frequency division multiple access (FDMA), code division multiple access (CDMA), and space division multiple access (SDMA) have been applied to BCIs using different EEG signals. In telecommunications, hybrid frequency and phase coding, which implements simultaneous frequency and phase coding in a multiple-access channel, has been proved more efficient than the frequency or phase only coding methods [10, 11]. The discriminability of SSVEPs could be improved by incorporating frequency and phase features in a similar way. Currently, hybrid frequency and phase coding for SSVEP-based BCIs has been rarely studied. Jia et al. [12] developed a mixed frequency and phase coding method to increase the number of classes in an SSVEP BCI. In their study, 15 targets were coded by 3 frequencies (10Hz, 12Hz, and 15Hz) and 4-6 phases (10Hz: 6 phases, 12Hz: 5 phases, 15Hz: 4 phases) under a 60Hz refresh rate. The system obtained an ITR of 66 bits/min in a simulated online test, showing the potential of hybrid frequency and phase coding in improving BCI performance. Since many applications of electroencephalogram (EEG)-based BCIs are greatly hindered by their communication speeds [9], it is of great importance to explore the capacity of hybrid frequency and phase coding in implementing a high-speed BCI.

Currently, a major challenge in implementing hybrid frequency and phase coding of SSVEPs is to present a large number of visual flickers on a computer monitor. The numbers of frequencies and phases that can be rendered on a monitor are always limited by the refresh rate since the number of frames in a stimulation cycle needs to be a constant and the number of phases can be realized is equal to the number of frames per cycle [12]. In this case, the mixed frequency and phase coding method can only realize a very limited number of classes. In [13], we proposed an approximation method to realize visual flickers with a high frequency resolution (e.g., 0.25Hz) using a computer monitor. Recently, we further proved that the phase of the SSVEPs elicited by the approximation approach was stable across different frequencies [14]. Therefore, the approximation approach can

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be extended to the phase domain, making it possible to implement hybrid frequency and phase coding with a large number of classes.

This study aimed to explore the feasibility and capacity of hybrid frequency and phase coding in multi-target SSVEP-based BCIs. Specifically, using a 40-target BCI speller, this study compared two hybrid coding strategies: (1) mixed frequency and phase coding, and (2) joint frequency and phase coding. At a spelling speed of 40 characters per minute (1.5 seconds per character), both coding approaches were tested using a simulated online BCI experiment. The goal of this study was to test the feasibility of hybrid frequency and phase coding in implementing high-speed SSVEP BCIs.

II. METHOD

A. *Hybrid Frequency and Phase Coding*

Stimulus approximation approach: In the conventional frame-based stimulus design, the number of frames in a stimulation cycle needs to be a constant. For each frequency, the number of phases can be realized is equal to the number of frames per cycle. For instance, with a 60Hz refresh rate, a 10Hz stimulus can be realized by reversing the stimulus pattern between black and white every three frames. For phase coding at 10Hz, six different phases (corresponding to six frames in a cycle) can be realized with a phase interval of 60 degrees. Using this method, a flickering frequency by which the refresh rate is not dividable (e.g., 11Hz) cannot be realized. In addition, an initial phase, which is not corresponding to a whole-number multiple of a single frame, cannot be realized too. To solve this problem, recently, we proposed an approximation approach that can reliably generate stimulus signals with flexible frequencies and phases [13, 14]. In general, the stimulus sequence $c(f,\emptyset,i)$ corresponding to frequency f and phase $\emptyset$ can be generated by the following equation:

$$c(f,\emptyset,i)=\frac{1}{2}\{1+\sin[2\pi f(i/Refreshrate)+\emptyset]\} \qquad (1)$$

where $\sin()$ generates a sine wave, and i indicates the frame index in the stimulus sequence.

Mixed frequency and phase coding: In mixed frequency and phase coding, all targets in a $K_x \times K_y$ stimulus matrix are specified by K_x phases (rows) and K_y frequencies (columns):

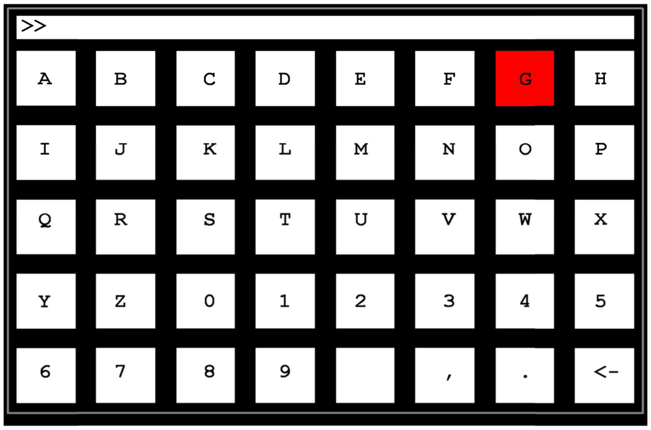
$$f(k_x,k_y)=f_0+\Delta f\times(k_y-1);$$
$$\emptyset(k_x,k_y)=\frac{2\pi}{K_x}\times(k_x-1);\quad (k_x=1{:}K_x,\ \ k_y=1{:}K_y) \qquad (2)$$

where k_x and k_y indicates the row and column index respectively, f_0 is the minimal stimulus frequency, Δf is the frequency interval between two adjacent columns, and $2\pi/K_x$ is the phase interval between two adjacent rows. Frequency and phase information has to be combined to identify a target when using mixed frequency and phase coding.

Joint frequency and phase coding: This study proposed a joint frequency coding method that can refine the widely used frequency coding approach [3-5]. In joint frequency and phase coding, two adjacent targets are tagged with different frequencies and different phases at the same time. Specifically, a total number of $K_x \times K_y$ targets from a stimulus matrix are tagged with linearly increasing frequencies and phases, of which the increments are both proportional to target index (from 1 to $K_x \times K_y$):

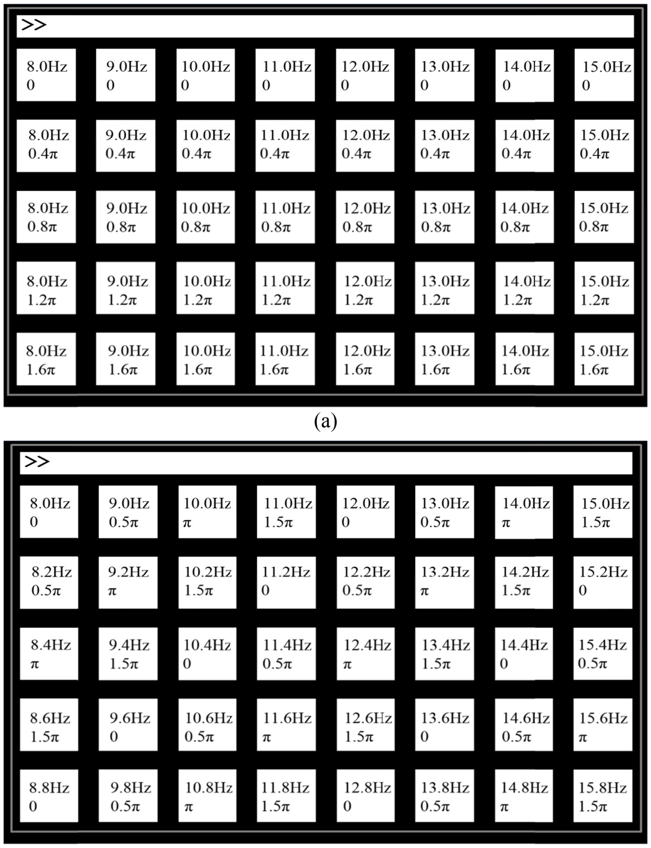
$$f(k_x,k_y)=f_0+\Delta f\times[(k_y-1)\times K_x+(k_x-1)];$$
$$\emptyset(k_x,k_y)=\emptyset_0+\Delta\emptyset\times[(k_y-1)\times K_x+(k_x-1)] \qquad (3)$$

where f_0 and $\emptyset_0$ indicate frequency and phase for the first target, Δf and $\Delta\emptyset$ represent frequency interval and phase interval between two adjacent stimuli. In this way, joint coding can incorporate frequency and phase coding to enhance the discriminability of multiple frequency-coded SSVEPs. In joint coding, a target can still be identified simply



A	B	C	D	E	F	G	H
I	J	K	L	M	N	O	P
Q	R	S	T	U	V	W	X
Y	Z	0	1	2	3	4	5
6	7	8	9		,	.	<-

Figure 1. The user interface of the 40-target BCI speller. The red square is the visual cue indicating a target character ‘G’ in the experiment.



8.0Hz 0	9.0Hz 0	10.0Hz 0	11.0Hz 0	12.0Hz 0	13.0Hz 0	14.0Hz 0	15.0Hz 0
8.0Hz 0.4π	9.0Hz 0.4π	10.0Hz 0.4π	11.0Hz 0.4π	12.0Hz 0.4π	13.0Hz 0.4π	14.0Hz 0.4π	15.0Hz 0.4π
8.0Hz 0.8π	9.0Hz 0.8π	10.0Hz 0.8π	11.0Hz 0.8π	12.0Hz 0.8π	13.0Hz 0.8π	14.0Hz 0.8π	15.0Hz 0.8π
8.0Hz 1.2π	9.0Hz 1.2π	10.0Hz 1.2π	11.0Hz 1.2π	12.0Hz 1.2π	13.0Hz 1.2π	14.0Hz 1.2π	15.0Hz 1.2π
8.0Hz 1.6π	9.0Hz 1.6π	10.0Hz 1.6π	11.0Hz 1.6π	12.0Hz 1.6π	13.0Hz 1.6π	14.0Hz 1.6π	15.0Hz 1.6π

(a)

8.0Hz 0	9.0Hz 0.5π	10.0Hz π	11.0Hz 1.5π	12.0Hz 0	13.0Hz 0.5π	14.0Hz π	15.0Hz 1.5π
8.2Hz 0.5π	9.2Hz π	10.2Hz 1.5π	11.2Hz 0	12.2Hz 0.5π	13.2Hz π	14.2Hz 1.5π	15.2Hz 0
8.4Hz π	9.4Hz 1.5π	10.4Hz 0	11.4Hz 0.5π	12.4Hz π	13.4Hz 1.5π	14.4Hz 0	15.4Hz 0.5π
8.6Hz 1.5π	9.6Hz 0	10.6Hz 0.5π	11.6Hz π	12.6Hz 1.5π	13.6Hz 0	14.6Hz 0.5π	15.6Hz π
8.8Hz 0	9.8Hz 0.5π	10.8Hz π	11.8Hz 1.5π	12.8Hz 0	13.8Hz 0.5π	14.8Hz π	15.8Hz 1.5π

(b)

Figure 2. Frequency and phase values for all targets using (a) mixed frequency and phase coding and (b) joint frequency and phase coding.

by frequency detection. However, detection accuracy can be improved by incorporating the embedded phase information.

B. BCI Speller

This study designed a 40-target BCI speller using the two hybrid frequency and phase coding approaches. As shown in Fig. 1, the user interface is a 5×8 stimulus matrix containing 40 characters (26 English alphabets, 10 digits, and four other symbols). Stimuli were presented on a 23.6-inch LCD screen with a resolution of 1920×1080 pixels using a 60Hz refresh rate. Each stimulus was presented within a 140×140 pixels square and the distance between two adjacent stimuli was 50 pixels. The stimulus program was developed under MATLAB (MathWorks, Inc.) using the Psychophysics Toolbox [15].

Fig. 2 illustrates the frequency and phase values used for each target. As shown in Fig. 2(a), 40 targets were specified by eight frequencies (8-15Hz with a 1Hz interval) and five phases (0, 0.4π, 0.8π, 1.2π, and 1.6π) in the mixed frequency and phase coding paradigm. The jointing coding paradigm used 40 frequencies (8-15.8Hz with a 0.2Hz interval) and the phase interval between two adjacent frequencies was 0.5π (Fig. 2(b)). Note that optimal selection of phase interval is out of the scope of this paper and will be investigated in another study.

C. Data Acquisition

Six healthy subjects (4 females, aged 25-27 years) with normal or corrected-to-normal vision participate in the experiment. All participants were asked to read and sign an informed consent form before participating in the experiment.

EEG data were acquired using a Synamps2 system (Neuroscan, Inc.) at a sampling rate of 1000 Hz. Nine electrodes (Pz, PO5, PO3, POz, PO4, PO6, O1, Oz, and O2) were placed over parietal and occipital areas according to the international 10-20 system. The reference electrode was placed at the vertex. Electrode impedances were kept below 10 kΩ. During the experiment, subjects were seated in a comfortable chair in a dimly lit soundproof room at a viewing distance of approximately 70 cm from the monitor.

This study designed a simulated online BCI experiment [12]. For each subject, 10 mixed-coding blocks and 10 joint-coding blocks were interleaved, resulting in a total of 20 blocks. Each block contained 40 trials corresponding to all 40 targets indicated in a random order. Each trial lasted 1.5 seconds, including 1 second for visual stimulation and 0.5 second for gaze shifting. Each trial began with a visual cue (a red square, see Fig. 1) indicating a target stimulus. Subjects were asked to shift their gaze to the target as soon as possible within 0.5 second. At 0.5 second after the cue onset, all 40 stimuli started to flicker for 1 second concurrently. To facilitate visual fixation, a red triangle appeared below the flickering target during the stimulation period.

D. Data Analysis

The EEG data were first down-sampled to 250Hz. For each target, 10 trials were extracted according to event triggers generated by the stimulus program, resulting in 400 trials for each coding approach. In each trial, 9-channel SSVEP data epochs corresponding to the 1-second stimulating duration were extracted for target identification. All data epochs were band-pass filtered from 7Hz to 70Hz using an infinite impulse response (IIR) filter.

This study adopted an extended canonical correlation analysis (CCA)-based method for target identification [16]. Training SSVEP reference signals $\hat{\boldsymbol{X}}$ can be obtained by averaging multiple trials in a training set. Correlation coefficient between projections of test set $\boldsymbol{X}$ and training reference signals $\hat{\boldsymbol{X}}$ using CCA-based spatial filters can be used to identify a target. Specifically, three canonical coefficients including (1) $\boldsymbol{W_X}(\boldsymbol{X}\hat{\boldsymbol{X}})$ between test set $\boldsymbol{X}$ and training reference signals $\hat{\boldsymbol{X}}$, (2) $\boldsymbol{W_X}(\boldsymbol{XY})$ between test set $\boldsymbol{X}$ and sine-cosine reference signals $\boldsymbol{Y}$, and (3) $\boldsymbol{W_X}(\hat{\boldsymbol{X}}\boldsymbol{Y})$ between training reference signals $\hat{\boldsymbol{X}}$ and sine-cosine reference signals $\boldsymbol{Y}$ are used as spatial filters for improving the SNR of SSVEPs. The target can be identified by recognizing the training reference signal that maximizes the correlation coefficient. Although the standard CCA-based method cannot discriminate different phases, the canonical correlation between $\boldsymbol{X}$ and $\boldsymbol{Y}$ still contributes to frequency detection. A correlation vector ρ is defined as follows:

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{bmatrix} = \begin{bmatrix} \rho(\boldsymbol{X}^T\boldsymbol{W_X}(\boldsymbol{XY}), \boldsymbol{Y}^T\boldsymbol{W_Y}(\boldsymbol{XY})) \\ \rho\big(\boldsymbol{X}^T\boldsymbol{W_X}(\boldsymbol{X}\hat{\boldsymbol{X}}), \hat{\boldsymbol{X}}^T\boldsymbol{W_X}(\boldsymbol{X}\hat{\boldsymbol{X}})\big) \\ \rho\big(\boldsymbol{X}^T\boldsymbol{W_X}(\boldsymbol{XY}), \hat{\boldsymbol{X}}^T\boldsymbol{W_X}(\boldsymbol{XY})\big) \\ \rho\big(\boldsymbol{X}^T\boldsymbol{W_X}(\hat{\boldsymbol{X}}\boldsymbol{Y}), \hat{\boldsymbol{X}}^T\boldsymbol{W_X}(\hat{\boldsymbol{X}}\boldsymbol{Y})\big) \end{bmatrix} \tag{4}$$

where $\rho(a, b)$ indicates the correlation coefficient between a and b. An ensemble classifier can be used to combine decisions from the four methods described above. In practice, the following weighted correlation coefficient $\tilde{\rho}$ is used as the final feature for target identification:

$$\tilde{\rho} = \sum_{i=1}^{4} \mathrm{sign}(\rho_i) \cdot \rho_i^2 \tag{5}$$

where sign() is used to remain discriminative information from negative correlation coefficients. The training reference signal that maximizes the weighted correlation value is selected as the reference signal corresponding to the target.

E. Performance Evaluation

This study used a leave-one-out cross-validation to estimate BCI performance in the simulated online experiment. In target identification, training reference signals were obtained from the training data in cross validation. This procedure was performed on mixed-coding and joint-coding datasets separately. Classification accuracy and simulated online ITR [12] were used for a direct comparison between the two methods. To further explore the interaction between BCI performance and stimulation frequency, this study also calculated the accuracy for each stimulation frequency.

III. RESULTS

Table I lists the classification accuracy and simulated online ITR for all subjects. The ITRs for both approaches (mixed coding: 172.37 bits/min; joint coding: 170.94 bits/min) were significantly higher than ITRs reported in previous SSVEP BCIs [2, 12]. To be noticed, subjects S2 and S4 obtained ITRs around 200 bits/min. ITRs were relatively stable across subjects (mixed coding: 139.54-207.53 bits/min; joint coding: 130.34-200.85 bits/min). For classification accuracy and ITR, there was no statistically significant difference between the two approaches (paired t-test, p>0.05).

TABLE I. SIMULATED ONLINE BCI PERFORMANCE

Subject	Accuracy (%)		ITR (bits/min)	
	Mixed	Joint	Mixed	Joint
S1	81.00	92.25	144.65	180.76
S2	98.25	97.50	204.09	200.85
S3	88.75	87.75	168.80	165.52
S4	99.00	97.25	207.53	199.80
S5	79.25	76.00	139.54	130.34
S6	89.00	82.25	169.62	148.37
Mean	89.21±8.30	88.83±8.57	172.37±28.67	170.94±28.32

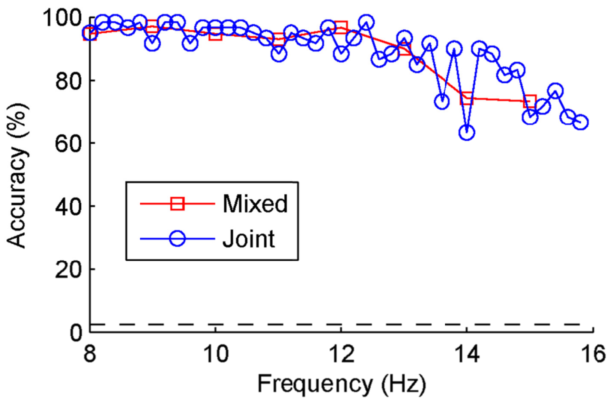


Figure 3. Averaged classification accuracy for each stimulation frequency across all subjects. In mixed coding, accuracy was averaged across five phases for each frequency. The dashed line indicates the chance level (2.5%) in target identification.

Fig. 3 illustrates classification accuracy for each stimulation frequency. The accuracy at each frequency was significantly higher than the chance level (2.5%). For both coding methods, the classification accuracy decreased as stimulation frequency increased. These results were consistent to the finding that SSVEPs in higher frequency band show lower amplitude response and SNR [5]. One-way analysis of variance (ANOVA) indicated significant difference between stimulation frequencies (mixed coding: $F(7,40)=3.11$, $p<0.05$; joint coding: $F(39,200)=2.05$, $p<0.001$).

IV. CONCLUSIONS AND DISCUSSIONS

This study investigated the feasibility of two hybrid frequency and phase coding methods in the implementation of a 40-target SSVEP-based BCI. At a spelling speed of 40 characters per minute (1.5 seconds per character), both approaches achieved high ITRs above 170 bits/min (mixed coding: 172.37±28.67 bits/min; joint coding: 170.94±28.32 bits/min). This study also showed that the joint coding approach obtained high ITR and classification accuracy comparable to the mixed coding approach. Taken together, this study suggests that the hybrid frequency and phase coding methods can provide a practical solution to implement a high-speed BCI speller.

To our knowledge, the joint frequency and phase coding approach has not been reported in SSVEP-based BCIs. In this study, the proposed joint coding approach showed high BCI performance comparable to the mixed coding approach [12]. Compared with the mixed coding method, the joint coding method might have several advantages. First, the joint coding method is a special case of the widely used frequency coding method. Therefore, the system still can work simply based on frequency detection when training data are not available. Second, the joint coding could be less sensitive to latency variation in SSVEPs, especially at higher stimulation frequencies. In contrast, in mixed coding, a latency variation could significantly deteriorate the discrimination of two adjacent targets at the same frequency.

Although the present study achieved very high ITRs, there is still room for improvement. The combination of frequency and phase resolutions could be further optimized for different frequency ranges separately. Another future direction is to compare the present method to the code modulation approach [17], which could be useful for developing more efficient hybrid coding methods for VEP-based BCIs.

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