Estimating Blood Pressure using Windkessel Model on Photoplethysmogram

Anirban Dutta Choudhury, Rohan Banerjee, Aniruddha Sinha and Shaswati Kundu

Abstract— **Simple and non-invasive methods to estimate vital signs are very important for preventive healthcare. In this paper, we present a methodology to estimate Blood Pressure (BP) using Photoplethysmography (PPG). Instead of directly relating systolic and diastolic BP values with PPG features, our proposed methodology initially maps PPG features with some person specific intermediate latent parameters and later derives BP values from them. The 2-Element Windkessel model has been considered in the current context to estimate total peripheral resistance and arterial compliance of a person using PPG features, followed by linear regression for simulating arterial blood pressure. Experimental results, performed on a standard hospital dataset yielded absolute errors of 0.78 ± 13.1 mmHg and 0.59 ± 10.23 mmHg for systolic and diastolic BP values respectively. Results also indicate that the methodology is more robust than the standard methodologies that directly estimate BP values from PPG signal.**

I. INTRODUCTION

Affordable health care in developing countries and elderly health care in both developing and developed countries require unobtrusive, low-cost devices to estimate physiological parameters. Blood pressure (BP) is the pressure exerted by circulating blood, upon the walls of blood vessels. During each heartbeat, blood pressure varies between a maximum (systolic; P_s) and a minimum (diastolic; P_d) value. Systolic blood pressure is the pressure exerted in the arteries when the heart contracts, whereas diastolic blood pressure is the pressure when the heart relaxes. A very high BP (hypertension) is an indicator of potential heart attack, arterial damages and stroke. A very low BP (hypotension) on the other hand may also lead to heart damages, kidney failure and several other heart diseases. Thus a regular check up of Blood pressure is needed for a basic preventive health care.

Invasive BP monitoring is rather uncommon, and generally takes place in hospital setups [1]. Traditionally, a physician or a trained medical technician uses combination of sphygmomanometer and stethoscope, listens for Korotkoff sounds, and determines the blood pressure levels. Though the later is a non-invasive procedure, the requirement of skilled professionals makes the process costly, and in some cases, regular check-up may become unaffordable.

In the last decade, off-the-shelf digital sphygmomanometers have flooded consumer market, offering medically unskilled users an opportunity to measure blood pressure, at virtually zero recurring cost. Some of these devices are medical-grade and aggressive pricing have made them quite affordable. Very recently, researchers have looked into cuff-less non-invasive blood pressure measurement with some of the products claiming FDA approval.[1]

Photoplethysmography (PPG) is a simple, non-invasive technique to measure instantaneous blood flow in blood vessels [2]. Affordable commercial devices (pulse oximeter) are available in market that captures fingertip PPG signal cleanly in transmissive mode. PPG is commonly used for the measurement of heart rate (HR), heart rate variability (HRV) [3] and arterial oxygen saturation (SpO2).

Zhang et al. [4] used ECG as well as PPG waveforms to estimate BP levels based on the principle of pulse wave transition by measuring the time lapse between the peak of the ECG and the trough of the PPG waveform. Chandrasekaran et. al [5] have used two sensor signals - PPG and audio sound of heart beat for BP measurement. A linear regression based model is created to estimate the systolic and diastolic BP using the time difference between the peak of the audio wave and the peak of the PPG signal. However, the major challenge of this system lies in synchronizing the two separate sensors (audio recorder and PPG recorder).

Few research works have tried to estimate P_s and P_d using PPG as their only input. Teng et al. [6] presented certain PPG features, for estimating human BP using linear regression. Lamonaca et al. [7] also proposed similar approach using neural network. Although these methods reported satisfactory accuracy, they do not physiologically relate PPG features with the values of P_s and P_d. On the other hand, there are several electrical [8] and mechanical [9] models available in literature that accurately simulate pressure wave propagation through arteries. In this paper, we have targeted the problem of estimation of P_s and P_d using PPG data through 2-Element Windkessel model [8]. PPG features of a subject are used to derive various latent parameters of Windkessel model that controls human BP.

Rest of the paper is organized as follows, BP wave simulation using 2-Element Windkessel model is briefly described in Section II. Section III deals with our proposed methodology to estimate BP, followed by experimental results and conclusion in Section IV and V respectively.

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[1] http://www.medgadget.com/2013/10/fda-approves-visi-mobile-system-for-cuffless-non-invasive-continuous-bp-monitoring.html

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II. Simulating Arterial BP using Windkessel Model

A. Windkessel Model

The Windkessel model describes the human cardiovascular system in terms of an electrical circuit [8]. The model can mathematically relate blood flow and blood pressure in arteries. In this analogy, arterial blood flow is described as the flow of fluid through a pipe. In the simplest form of Windkessel model (2-Element Windkessel model), the total peripheral resistance and arterial compliance are modeled as a resistance (R in $mmHg.s/mL$) and capacitance (C in $mL/mmHg$) respectively. The blood flow from ventricles to artery is analogous to a sinusoidal electrical current ($I(t)$) wave and arterial pressure wave is modeled as a time-varying electrical potential ($P(t)$).

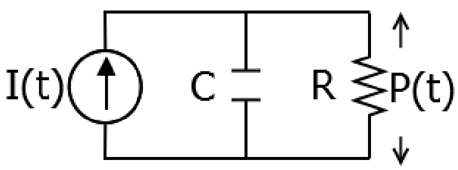


Fig. 1: Electrical analogy of 2-Element Windkessel model

As the number of elements increase (3 or 4 Element model), the accuracy of the model increases with an additional penalty of more unknown variables to deal with. For simplicity, we have used 2-Element Windkessel model in our proposed methodology to reduce the number of unknown quantities to be predicted. The electrical analogy of 2-Element Windkessel model is shown in Fig. 1. Applying Kirchhoff Law of Current, we get

$$\frac{P(t)}{R} + C\frac{\mathrm{d}P(t)}{\mathrm{d}t} = I(t) \tag{1}$$

B. Derivation of P_s and P_d

Without loss of generality, the total blood pumped by heart during n^{th} cardiac cycle can be expressed (as in Eq.2) as a sinusoidal function with a peak value of I_0 during systole and *zero* during diastole.

$$I(t) = \begin{cases} I_0 sin(\frac{\pi t}{T_s}), & (n-1)T_c < t \leq (n-1)T_c + T_s \\ 0, & (n-1)T_c + T_s < t \leq nT_c \end{cases} \tag{2}$$

Here T_s is the *systolic upstroke time*, T_d is the *diastolic time*

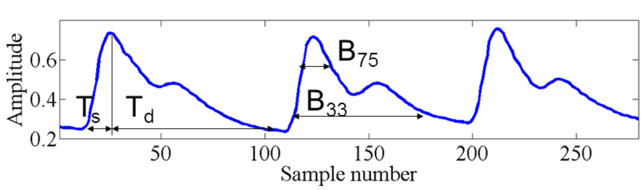


Fig. 2: Sample PPG waveform.

and the duration of one *cardiac cycle* is $T_c = T_s + T_d$. If C_o be the cardiac output of a person[2], then for one cardiac cycle

$$\frac{C_o T_c}{60} = I_0 \int_0^{T_s} sin(\frac{\pi t}{T_s})\mathrm{d}t \tag{3}$$

$$I_0 = \frac{C_o T_c}{60 \int_0^{T_s} sin(\frac{\pi t}{T_s})\mathrm{d}t} \tag{4}$$

Now, putting the conditions of Eq.2 in Eq.1 and solving them, we can form an expression for $P(t)$, which brings us the following:

$$\begin{aligned} P_s &= P(t|t = T_s) \\ &= P_{ts}e^{-T_s/RC} + \frac{I_0 T_s C\pi R^2}{T_s^2 + C^2\pi^2 R^2}(1 + e^{-T_s/RC}) \end{aligned} \tag{5}$$

$$P_d = P(t|t = T_d) = P_{td}e^{-T_d/RC} \tag{6}$$

Where P_{ts} and P_{td} are the initial values of P_s and P_d respectively. Similarly, R and C can also be expressed as function of P_s, P_d, T_s, T_d and I_0 using Eq.5 and Eq.6 respectively.

C. Inputs and Assumptions

We assume that every cardiac cycle starts at systole and P_{ts} is supplied as 80 mmHg as an initial condition for the first cycle. For every next cycle this gets replaced by P_d calculated in the previous cycle. P_{td} is assigned to the value of P_s obtained in that cycle. The ventricular blood flow and BP waveform of an imaginary subject having heart-rate of 90 bpm, P_s and P_d of 160 mmHg and 100 mmHg respectively are shown in Fig. 3. It can be observed that after few cardiac cycles the waveform gets stabilized to the desired BP levels. For this example, we have assumed that $T_d/T_s = 1.5$. Thus the model is capable of simulating any BP waveform, provided R, C, T_s and T_d are known.

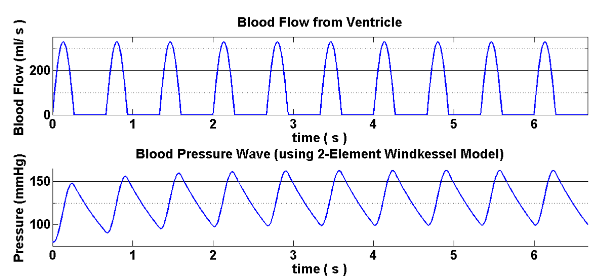


Fig. 3: BP simulation using 2-Element Windkessel model for P_s/P_d = 160/100 mmHg and HR = 90 bpm

As shown in Fig.2 parameters like T_s, T_d and T_c can be accurately calculated from the input PPG signal. For a healthy adult person cardiac output can be well-assumed to be 5 litre/min. So I_0 can be calculated from this using Eq.4. However there is no known mathematical relationship between PPG features and R, C. Physiologically, the R and C components affect the blood flow in arteries and are expected to be related with the shape of the PPG waveform of the subject. So we use machine learning based approach to estimate R and C from PPG features.

[2]units of C_o and T_c are $litre/minute$ and $second$ respectively

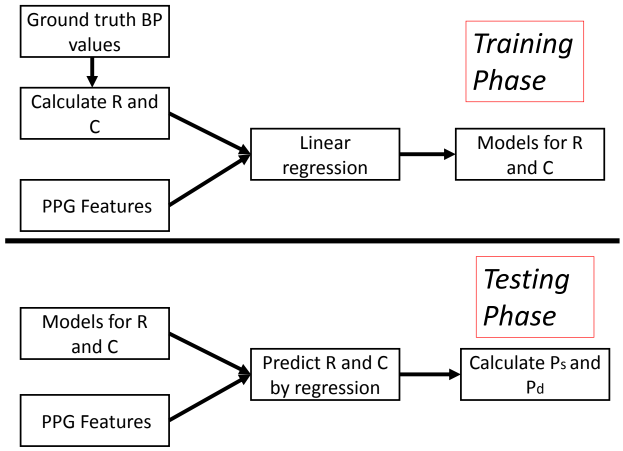


Fig. 4: Block diagram of proposed methodology

In our proposed methodology (as in Fig. 4), R and C of the training subjects are first calculated from ground truth BP values and are fitted to a straight line using their PPG features in the off-line training phase. During testing, R and C are first estimated from PPG features of the test subject and later P_s and P_d are calculated using Eq.5 and Eq.6.

III. Proposed Methodology

Different steps involved in analysing PPG signal for feature extraction, estimation of R, C parameters and later P_s and P_d are given in the following subsections.

A. Feature Extraction from PPG signal

Generally a PPG signal is noisy in nature and may contain a DC component along with some high frequency noise components. The fundamental frequency of PPG signal is typically concentrated around 1 Hz (depending upon the heart rate of the subject). So a bandpass filter having cut-off frequencies of 0.7 Hz and 3 Hz are used to remove the undesired frequency components form PPG signal as part of preprocessing. Later, a set of time domain features are extracted from each cycle of PPG signal. In our case, initially we have considered all the features mentioned in [6] and [7]. Applying the Maximal Information Coefficient (MIC) based feature selection technique mentioned in [10], our optimum feature set gets reduced to the following - (1) systolic upstroke time (T_s), (2) diastolic time (T_d), sum of systolic width and diastolic width at (3) 33% (B_{33}) and (4) 75% (B_{75}) of pulse amplitude for R and C. Heart rate of the subject is also considered as a feature for C.

B. Outlier Removal

Feature extraction requires an accurate detection of the peak and trough points from each cycle of the input PPG data. A misdetection or a false detection of peak or trough points can lead to wrong feature calculation. Generally the wrongly detected features (outliers) are either too large or too small compared to the actual range of the feature values. In our case, we use the threshold based approach mentioned in [11] to successfully remove the outlier feature data before regression analysis.

C. Estimation of R and C from PPG features

R and C parameters are estimated using multiple linear regression where the PPG features are treated as independent variables. In training phase, the regression models are fitted using least square method to calculate the regression parameters. These parameters are used for estimation of R and C of an untrained subject in the testing phase.

D. Estimation of BP values

T_s and T_d can be calculated from PPG waveform. Once R and C are estimated, BP values of the subject are calculated using Eq.5 and Eq.6 respectively.

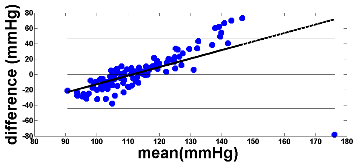
IV. Experimental Results

All our experiments were conducted on The University of Queensland Vital Signs Dataset [12]. It is a standard hospital dataset containing simultaneously recorded PPG and BP data for 32 surgical cases, ranging in duration from 13 minutes to 5 hours over a period of 4 weeks, using standard medical devices. The dataset also contains the fluctuation of BP values of a person over a long period of time and hence is an ideal dataset for performance analysis of our proposed methodology. The pleth data is sampled at 100 Hz. However, we neglect the instantaneous fluctuation of P_s and P_d and take their arithmetic mode over a period of 5 minutes to return a single value for annotation. The dataset is split into two halves of equal size, one for training and the other for performance evaluation. The training set is ensured to contain a wide variation of P_s and P_d values in almost equal distribution in order to create unbiased training models.
The performance of our proposed methodology is tested against the two methods mentioned in [6] and [7]. Linear regression is used as the machine learning tool for all the three methods for unbiased performance analysis. The absolute error from ground truth for both P_s and P_d are reported in the manner of $mean \pm std$ in Table I for performance comparison. Results clearly indicate that our proposed methodology outperforms methods [6] and [7].

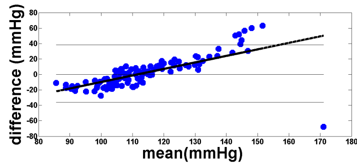
TABLE I: Performance comparison for P_s and P_d in absolute error ($mean \pm std$) and slope of the best fitted straight line in respective Bland-Altman plots

	P_s		P_d	
Method	P_s (mmHg)	Slope of B-A plot forP_s	P_d (mmHg)	Slope of B-A plot forP_d
Method [6]	1.5 ± 23.3	0.83	1.88 ± 16.6	1.75
Method [7]	1.17 ± 19.1	0.7	1.3 ± 13.6	1.2
Our Method	0.78 ± 13.1	0.42	0.59 ± 10.2	0.52

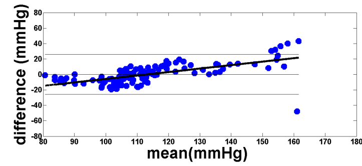
Bland-Altman plot [13] is a well-known method for assessing agreement between two methods of clinical measurements. In Fig. 5 and Fig. 6, we have presented the agreements of each of the three methods ([6], [7] and our proposed methodology) against the clinically measured BP values. In all the plots, the horizontal axis indicates the mean of the estimated and clinical BP ground truth values, whereas the



(a) Method [6]

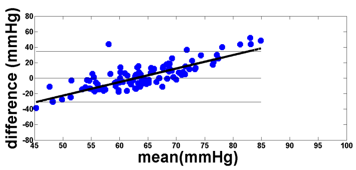


(b) Method [7]

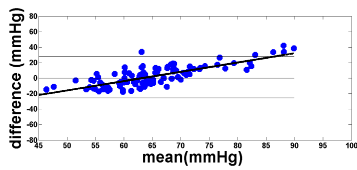


(c) Proposed method

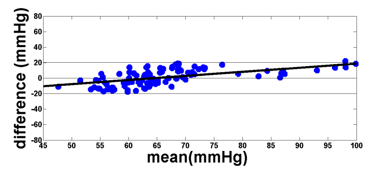
Fig. 5: Bland-Altman plot for measured P_s w.r.t Ground truth



(a) Method [6]



(b) Method [7]



(c) Proposed method

Fig. 6: Bland-Altman plot for measured P_d w.r.t Ground truth

vertical axis represents the difference between them. Hence, a horizontal spread is desired in the Bland-Altman plots, as it confirms that the concerned methods were tested on varied range of dataset. Conversely, a vertical spread is not desired as it would indicate the disagreement with clinical BP values.

We have also plotted the best linear fit for each of the Bland-Altman plots. A higher slope indicates a higher disagreement. A very small slope (≈ 0) of the best fitted straight line in a Bland-Altman plot would indicate that the respective methods closely match with each other. As shown in Table I, our proposed method yields lowest slopes for both P_s and P_d, and hence we claim that the proposed methodology outperforms the other two methods, and matches more closely with clinical method.

In all our cases, we have made an assumption of cardiac output of each subject as 5 litre/minute. As the current dataset holds a number of critical surgical cases, there can be instances where subjects having cardiac output lesser than that. For such cases there is a chance that the estimated BP values deviate much from the ground truth. Currently, we are exploring some non-invasive methods to coarsely estimate the cardiac output of a person.

V. Conclusion

A photoplethysmographic approach to estimate human blood pressure using 2-Element Windkessel model has been presented in the current context. Experimental results show that the methodology of estimating BP via the intermediate latent parameters of R and C can improve the overall accuracy. However, 2-Element Windkessel model does not take into account many internal parameters that affect human BP. Thus, the method still needs to be validated against 3 or 4-Element Windkessel models along with other standard models available in literature that simulate human BP more accurately. The present algorithm is tested on hospital PPG data, captured using standard medical device which is expected to be noise-free. Our future challenge lies in running the same algorithm successfully on noisy reflective PPG data captured by smart phones, to make the entire system available as a phone application for BP monitoring.

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