Synthetic Optimization of Air Turbine for Dental Handpieces

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***Abstract*—A synthetic optimization of Pelton air turbine in dental handpieces concerning the power output, compressed air consumption and rotation speed in the mean time is implemented by employing a standard design procedure and variable limitation from practical dentistry. The Pareto optimal solution sets acquired by using the Normalized Normal Constraint method are mainly comprised of two piecewise continuous parts. On the Pareto frontier, the supply air stagnation pressure stalls at the lower boundary of the design space, the rotation speed is a constant value within the recommended range from literature, the blade tip clearance insensitive to while the nozzle radius increases with power output and mass flow rate of compressed air to which the residual geometric dimensions are showing an opposite trend within their respective "pieces" compared to the nozzle radius.**

I. Introduction

Pelton turbine using kinetic energy from expansion of compressed air has been applied in dental procedures like cutting, reshaping, etc. for over 40 years[1][2]. Several studies have been carried out to investigate the characteristics of its performance under real working conditions. Dyson and Darvell[3] implemented experiments, which covers a wide range of handpiece designs, for analyzing functional relationship between vital variables to the turbine performance, i.e. gas property, supply pressure, volumetric flow rate, key geometric dimensions and free running speed. Based on a trial–and–error model selection process and linear regression, semi–empirical correlations are proposed for volumetric flow rate and pressure effectiveness aiming at a correlation for free rotation speed prediction. The authors[4] further explored the dependency of stall torque on supply air stagnation pressure, stall torque and bearing resistance on relative rotation position and dynamic torque and power output on rotation speed and supply air gauge pressure. Considering free rotation speed and stall torque at a suggested supply pressure, standardized power index is proposed, built up upon which efficiency index is further introduced by referring to the maximum power output achievable under an isentropic expansion assumption. Hwang et al.[5] illustrated a design process on a basis of ideal gas model and Euler's work equation for turbomachinery while at the exit the relative flow velocity is assumed to be zero. The inlet mass flow rate is corrected by incorporating the effect of clearance between blade tip and turbine casing. Also the positioning and cross section area of the supplying nozzle is mentioned in the design to maximize the energy conversion efficiency.

Previous researches on the performance of air turbine in dental handpieces usually focused on the impact of different variables on the performance or efficiency of dental handpieces. However, it is still difficult for the end user to choose an appropriate product since all nominal indexes indicating performance or efficiency are acquired under designated working conditions, e.g. supply air pressure, whereas in practice the facility is not always running with these preferred conditions for optimized performance or efficiency. Moreover, it is even complicated when two competing indexes exist. For instance, to increase the effectiveness of cutting process under a given torque load, higher rotating speed is required which brings higher supply air pressure and therefore the air consumption rate. In the present study, three competing variables, power output–P, supply air mass flow rate–$\dot{m}$ and rotation speed–ω of the turbine, are selected for simultaneous optimization meaning that instead of a single optimum solution set (including working condition and geometric design), the optimization process yields sets of solutions (Pareto solution sets) so that all competing variables cannot be further optimized simultaneously. More significantly, from the perspective of making a choice from products with various configurations, it is wiser for the end user to acquire the optimum solution sets as a standard which usually covers a wider range of geometric designs with corresponding working conditions.

II. Model Description

Fig. 1 shows all major geometric dimensions of the Pelton air turbine that have effect on the performance of dental handpieces, the turbine radius R_t, the converging nozzle radius R_n, the rotor core radius R_c, the turbine blade width w_b and the turbine blade tip clearance δ. Other impacting variables i.e. the supply air stagnation pressure p_0 (subscript "0" hereinafter indicates variable evaluated at the nozzle inlet) and the rotation speed ω are also shown in Fig. 1.

From Euler's turbine equation assuming a zero relative velocity at the exit of the blade, the specific power output is

$$P = \dot{m}_c U\left(c_i - U\right) \quad (1)$$

where the turbine rotation speed U is defined as follows.

$$U = R_m \omega \quad (2)$$

in which

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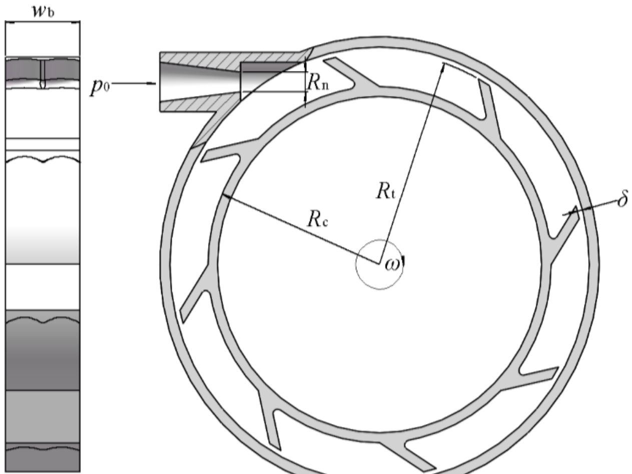

Figure 1. Schematic representation of Pelton air turbine in dental handpieces

$$R_m=(R_t+R_c)/2. \tag{3}$$

The corrected mass flow rate $\dot{m}_c$ considering the loss caused by blade tip clearance[5] is calculated by

$$\dot{m}_c = \frac{(R_t - R_c)\dot{m}}{(R_t - R_c + \delta)} \tag{4}$$

where $\dot{m}$ represents the exit–nozzle mass flow rate (before correction). According to mass conservation the jet velocity c_i (subscript "i" hereinafter indicates the variable evaluated before impinging the blade) is given by

$$c_i = \frac{\dot{m}}{\rho_i (R_t - R_c) w_b} \tag{5}$$

where ρ_i represents the air density which is given by the ideal gas model as

$$\rho_i = \frac{p_i}{R_{air} T_i} \tag{6}$$

in which R_{air} is the molecular gas constant of air. Assuming that the flow through the nozzle is adiabatic[3-6], the mass flow rate is further elaborated as

$$\dot{m}_i = \pi R_n^2 \sqrt{\frac{2\kappa}{\kappa - 1} p_0 \rho_0 \left[\left(\frac{p_i}{p_0} \right)^{\frac{2}{\kappa}} - \left(\frac{p_i}{p_0} \right)^{\frac{\kappa+1}{\kappa}} \right]} . \tag{7}$$

where κ is the isentropic expansion factor. In the present study the stagnation temperature of supply air T_0 is assumed to be 25°C, a normal room temperature. The corresponding air density can be obtained using (5) thereafter. The value of p_i is dependent on the exit–ambient–to–entry pressure ratio of the nozzle. When the pressure ratio is higher than the critical pressure ratio defined as

TABLE I. DESIGN SPACE FOR PELTON TURBINE IN DENTAL HANDPIECES

	Lower Boundary	Upper Boundary
R_t (m)	3.81E–3	4.46E–3
w_b (m)	1.56E–3	3.33E–3
R_c (m)	2.25E–3	2.61E–3
R_n (m)	2E–4	6E–4
δ (m)	1E–5	1.3E–4
ω (rad/s)	2.09E+04	$\pi N_f/30$
p_0 (Pa)	1.26E+05	6E+05

$$\beta = \left(\frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa - 1}} \tag{8}$$

On the exit plane, the air entering the nozzle will expand to the ambient pressure in the turbine casing (normally p_i=101325 Pa), and the exit flow will be subsonic. However, when this pressure ratio becomes lower than β, the pressure on the exit plane will be kept at βp_0 and the air flow further expands to the ambient pressure within the turbine chamber[6].

III. OPTIMIZATION

A. Objective Function

In dentistry practice it is always expected that the rotation speed of dental drill be faster so that the patients' pain and treatment time can be reduced[5]. In the meantime, since most of the power output will be dissipated to cause biological damage to the pulp and the mechanical damage to tooth structure due to thermal stress, not to mention those via bearing friction, loss in Pelton blade and cutting tooth or dental fillings, a lot of cooling water is therefore needed in a dental procedure[7][8]. Minimizing the power output should be considered in the design as well. Furthermore, the Pelton turbine in a dental handpiece is driven by the pressurized air of which the consumption rate is to be considered as well. However, from the functional relationship above, it can be inferred that all three objective functions P, $\dot{m}$ and ω are competing each other to some extent. Thus, a multi–objective optimization formulated below is required to tackle this problem.

$$\begin{aligned} &\min \quad P, \dot{m} \\ &\max \quad \omega \end{aligned} \tag{9}$$

subject to the free running speed limitation N_f as proposed in (19), [3]. All variables are limited in the design space as listed in Tab. I according to [3][5] which comprises a large scope of designs from different manufacturers.

B. Pareto Solution Generation

Several methods aiming at generation a set of Pareto solutions for either decision making or design optimization have been proposed. Among all these widely used methods, the Weighted Sum (WS) and the Compromising Programming (CP) method[9], share the same attributes of failing to generate well–distributed Pareto solution sets, which is believed to be important in either adequately representing the

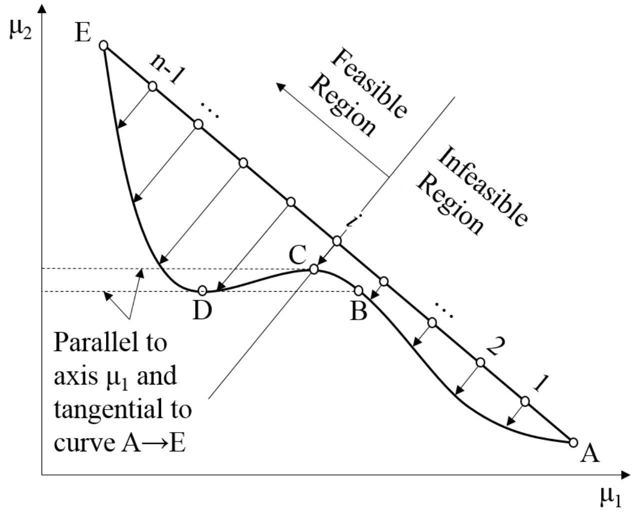

Figure 2. Graphical representation of NBI and NC method

entire design space or effectively exploring the design space by altering the aggregate objective function weights. Both the normal boundary intersection (NBI)[10] and the Normal Constraint (NC) method generate dominated or nonPareto solutions which the NBI method is more likely to generate due to its utilization of equality constraints. The Genetic Algorithm (GA)[11] and the later developed Evolutionary Algorithm (EA)[12] are also found to generate unevenly distributed Pareto solutions, probably due to their stochastic nature, although they claim to generate less dominated solutions[13].

For all the methods aforementioned, NBI and NC seem the most promising candidates since they are not flawed by the generation of ill–distributed Pareto sets given that the nonPareto solutions generated can be eliminated properly, which yields the Normalized Normal Constraint (NNC) method with Pareto filter [13] which is applied in the present study and discussed as follows.

For simplicity in illustration, a bi–objective case is addressed assuming that the problem is to minimize two objective functions, μ_1 and μ_2, simultaneously. The first step is to solve the single objective minimization problem regarding μ_1 and μ_2 individually using either gradient–based or stochastic algorithm. The acquired objective function values are respectively noted as μ_1^* and μ_2^* which will be normalized to a 0→1 scale by a linear transformation and also marked as E and A in Fig. 2. The line segment AE is then evenly divided in to n sections by n–1 nodes (node 0 = A and node n = E). In the NBI method, a single objective minimization respecting μ_2 will be implemented given a linear equality constraint going through node (and step) i (i is from 1 to n-1 sequentially) and perpendicular to line segment AE using the Pareto solution set from step i-1 as a initial guess. After a thorough traversing from node 1 to node n, a curve passing through A→E alphabetically (see Fig. 2) can be expected as the Pareto frontier. However, the curve is not exactly a Pareto frontier if it consists of both convex and concave parts as depicted in Fig. 2 (e.g. C is dominated by D). But if the NC method, which intentionally prescribes a feasible region by using a linear inequality constraint defined in the same way as described for the NBI method, is applied, the minimization will result in D in step i leaving a gap between C and D which is not a part of the expected Pareto frontier. Nevertheless, curve segment B→C (B excluded), which is obvious dominated by D, still needs elimination in a Pareto filter, which essentially keeps the nondominated element pairs ($\mu_1^{f,k}$, $\mu_2^{f,k}$) (1≤k≤number of elements in μ_1 or μ_2) in array μ_1 and μ_2 by an exhaustive comparison between any two element pairs (see Fig. 3).

IV. RESULT AND DISCUSSION

An NNC method with Pareto filter and sequential quadratic algorithm for each inner step is applied to (9) and results are shown in Fig. 2 and Tab. II with the median node and two end nodes (see Fig. 2) selected from the piecewise continuous part which dominates the Pareto frontier.

In Tab. II, the supply air stagnation pressure p_0 lies on its lower boundary indicating a lower supply air pressure for practical operation within the current design space under discussion. A rotation speed of 7.58E+04 rad/s (equivalently 7.24E+05 rpm) is found to be optimal which is consistent with the recommended value of above 6E+05 rpm in [5].

On the other hand, for geometric dimensions it can be seen that nozzle radius increases with power output and mass flow rate while rotor core radius, turbine radius and blade height decreases with increasing power output and mass flow rate within an individual piece (Set 1 → Set 3 or Set 4 → Set 6). There is no deterministic relationship between blade tip clearance and power output or mass flow rate.

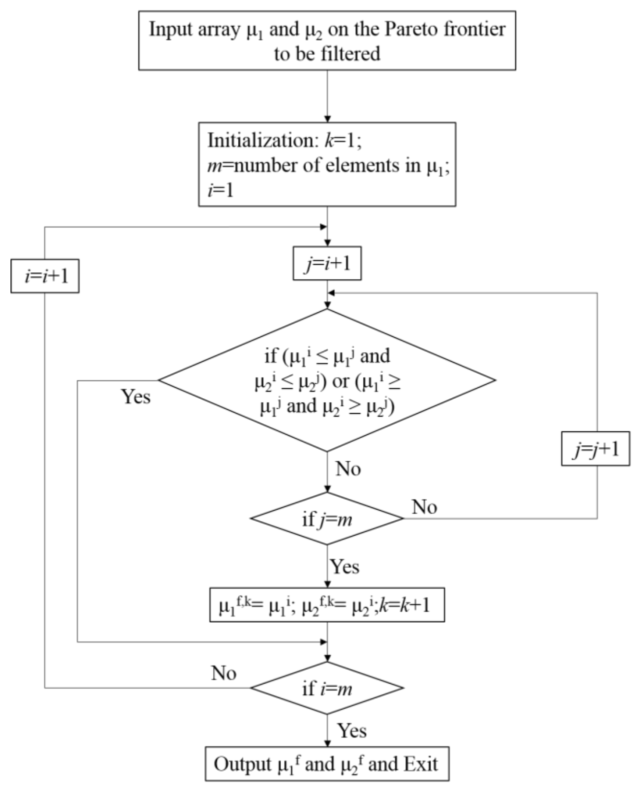

Figure 3. Flow chart for Pareto filter

TABLE II. SELECTED PARETO SOLUTION SETS

	R_n (m)	R_t (m)	R_c (m)	p_0 (Pa)	w_b (m)	δ (m)	P (W)	$\dot{m}$ (kg/s)	ω (rad/s)
Set 1	3.02E-04	3.90E-03	2.34E-03	1.26E+05	1.65E-03	1.34E-04	3.20	6.76E-05	7.58E+04
Set 2	3.23E-04	3.86E-03	2.30E-03	1.26E+05	1.61E-03	1.34E-04	3.50	7.72E-05	7.58E+04
Set 3	3.50E-04	3.80E-03	2.24E-03	1.26E+05	1.55E-03	1.38E-04	3.84	9.05E-05	7.58E+04
Set 4	3.53E-04	3.90E-03	2.34E-03	1.26E+05	1.65E-03	1.37E-04	4.21	9.22E-05	7.58E+04
Set 5	3.66E-04	3.86E-03	2.30E-03	1.26E+05	1.61E-03	1.37E-04	4.35	9.90E-05	7.58E+04
Set 6	3.81E-04	3.82E-03	2.26E-03	1.26E+05	1.57E-03	1.38E-04	4.50	1.07E-04	7.58E+04

V. CONCLUSION

A synthetic (including geometric dimension and working condition) design optimization, aiming at reducing power output, compressed air consumption and increasing rotation speed of the dental drill simultaneously, is implemented with a combination of model–based Pelton turbine design procedure and design space from existing industrial applications embedded. Within the current design space from literature which covers a wide range of products from different manufacturers, it is found that the optimal solution sets (Pareto frontier) majorly consists of two piecewise continuous parts in each of which nozzle radius increases with power output and mass flow rate whereas all the rest geometric dimensions concerned is showing an opposite trend except for the blade tip clearance which is far less sensitive to both power output and mass flow rate. On the other hand, working condition variables i.e. supply air stagnation pressure (close to lower limit of the design space) and rotation speed (consistent with the recommended value in practice) are held constant for all Pareto solution sets.

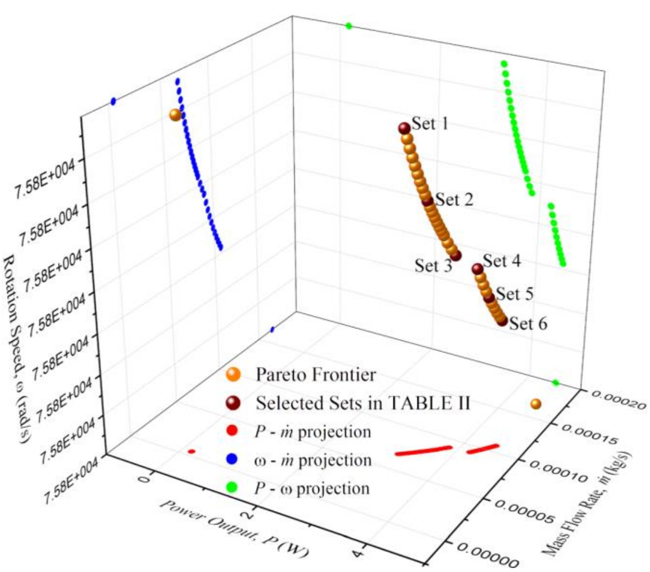

Figure 4. Pareto frontier for the design of Pelton turbine in dental handpieces

Experimental verification of the optimized design will be carried out in the next-phase work.

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