Estimation of Crank Angle for Cycling with a Powered Prosthesis

B. E. Lawson, A. Shultz, E. Ledoux, and M. Goldfarb[1]

Abstract— **In order for a prosthesis to restore power generation during cycling, it must supply torque in a manner that is coordinated with the motion of the bicycle crank. This paper outlines an algorithm for the real time estimation of the angular position of a bicycle crankshaft using only measurements internal to an intelligent knee and ankle prosthesis. The algorithm assumes that the rider/prosthesis/bicycle system can be modeled as a four-bar mechanism. Assuming that a prosthesis can generate two independent angular measurements of the mechanism (in this case the knee angle and the absolute orientation of the shank), Freudenstein's equation can be used to synthesize the mechanism continuously. A recursive least-squares algorithm is implemented to estimate the Freudenstein coefficients, and the resulting link lengths are used to reformulate the equation in terms of input-output relationships mapping both measured angles to the crank angle. Using two independent measurements allows the algorithm to uniquely determine the crank angle from multi-valued functions. In order to validate the algorithm, a bicycle was mounted on a trainer and configured with the prosthesis using an artificial hip joint attached to the seat post. Motion capture was used to monitor the mechanism for forward and backward pedaling and the results are compared to the output of the presented algorithm. Once the parameters have converged, the algorithm is shown to predict the crank angle within 15° of the externally measured value throughout the entire crank cycle during forward rotation.**

I. Introduction

As lower limb powered prostheses begin to emerge in the commercial market, amputees will likely desire to use these devices for activities outside of those necessary for everyday living. The majority of research on powered prostheses focuses on the mobility and stability benefits of walking on level ground, slopes, and stairs [1–5]. Cycling, however, is both a popular recreation and also a tool used for fitness and rehabilitation. It is also an activity that is characterized by significant net power generation at the hip, knee, and ankle joints [6–9]. It can be predicted, then, that lower limb amputees would suffer significant performance disadvantages when using passive prostheses for cycling. Some recent work has been done exploring cycling in transtibial amputees (with passive ankles), but the authors know of no comparable studies for transfemoral amputees [10]. This manuscript marks the beginning of an investigation into what is necessary for a powered knee and ankle prosthesis to contribute power during cycling in transfemoral amputees.

In healthy biomechanics, the majority of the external work done by the pedaling limb is performed during what is known as the power stroke. The power stroke consists of knee and hip extension and ankle plantarflexion when the crank arm is in the forward half of its revolution. If the angle of the crank is denoted by θ_c and the convention shown in Fig. 1 is adopted, then the power stroke occurs between approximately 30° and 120° for the particular configuration shown. The power stroke region will generally vary as a function of the seat tube angle (φ_0 in Fig. 1), as the mechanical advantage of the lower limb joints with respect to the crank is a function of their relative angles and not their orientation with respect to gravity. For example, the power stroke for a recumbent bicyclist would be approximately 90° earlier than for an upright bicyclist.

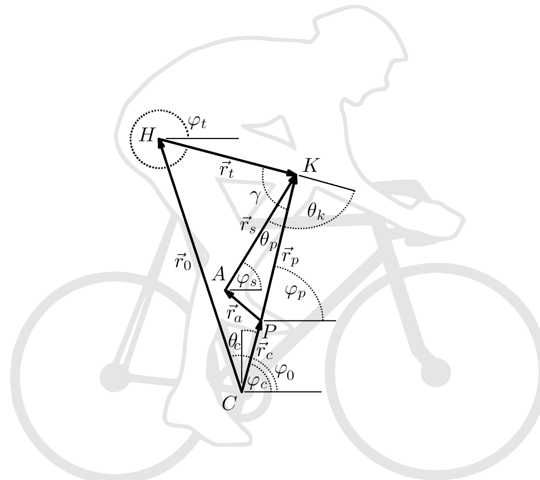

Fig. 1: Kinematic diagram of the four bar linkage model.

If a powered prosthesis is going to supply torque to supplement an amputee's effort during cycling, an estimate of the crank angle is critical. The crank cycle is generally divided into 4 strokes: top, power, bottom, and recovery [10]. The transmission of torque from the knee to the crank inverts during the top and bottom strokes. The crank angle, along with the direction of rotation, must be known to the prosthesis to avoid supplying an extensive torque during the recovery stroke or a flexive torque during the power stroke. At the very minimum, therefore, an estimator is needed that can determine the initiation and termination of the power and recovery strokes.

II. Methods

It is assumed that the prosthesis can measure θ_k and φ_s. For the powered prosthesis previously developed by the authors, θ_k is measured by an absolute magnetic encoder, while φ_s is measured by an inertial measurement unit (IMU).

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With these two measurements, the relative lengths of each link in the four bar mechanism can be determined, and the result can then be used to uniquely determine the crank angle.

A. Estimation of Link Lengths

For the purposes of this work, the rider/prosthesis/bicycle system is assumed to be appropriately modeled by a planar four-bar linkage. This model therefore assumes that (1) the hip joint remains in a fixed location relative to the crank axis and (2) that the ankle joint is capable of remaining infinitely stiff such that the shank and foot can be treated as a single link. Under these conditions, this single degree-of-freedom mechanism can be completely characterized by the generalized coordinate φ_c, which is the angle of the crank shaft with convention as shown in Fig. 1. Note that this convention is different from that generally used in the biomechanics literature (denoted in Fig. 1 by θ_c), which defines the top-dead-center (TDC) position of the crank arm as zero, with forward rotation being positive [6, 10]. Because this is a single degree-of-freedom system, φ_c uniquely determines the configuration of the system under the further assumption that the knee cannot hyper-extend. (If hyperextension were allowed, there would be two assembly modes for the mechanism.) However, this mapping is a function of the relative link lengths of the mechanism, and so these link lengths must be determined if the relationship between φ_c and an internal prosthesis measurement, such as the knee angle, θ_k, is going to be exploited to determine the crank angle.

In general, it would be best to avoid specifying the geometry of the system explicitly since these parameters will likely change between riders and bicycles. Therefore, only the following parameters are specified: the seat tube angle, φ_0 (for most bicycles, this is close to 75°), r_p, and θ_p. With these parameters set and assuming at least 3 known input and output angles of the mechanism, the link lengths can be determined uniquely through classic analytical methods. If φ_p and φ_t are used (which can be uniquely determined from the measured angles θ_k and φ_s), then Freudenstein's equation can be written in the following form (derived from the loop close equation: $\vec{r}_c + \vec{r}_p = \vec{r}_0 + \vec{r}_t$).

$$K_1 \cos\varphi_{t0} + K_2 \cos\varphi_{p0} + K_3 = \cos\varphi_{tp} \quad (1)$$

where the notation φ_{ab} denotes $(\varphi_a - \varphi_b)$. The coefficients are given by

$$\begin{aligned} K_1 &= \frac{r_0}{r_p} \\ K_2 &= -\frac{r_0}{r_t} \\ K_3 &= \frac{r_0^2 + r_t^2 + r_p^2 - r_c^2}{2 r_p r_t} \end{aligned} \quad (2)$$

The orientation of the shank with respect to gravity (and, through the knowledge of φ_0, also with respect to the bicycle frame representing the fixed link of the four bar mechanism) is estimated in real time by combining the high frequency portion of the integral of the in-plane angular rate measured by a solid state gyroscope with the low frequency portion of the inverse tangent of the in-plane accelerometer signals through the use of first order complementary filters with time constants of one second. The orientation of the thigh is determined by adding the knee angle (less θ_p) to the shank orientation. As the prosthesis moves through the cycling motion (initially generated, at least, by effort from the hip or the contralateral limb), pairs of angles are continually generated that should be consistent with the geometry of a particular four-bar mechanism. A continuous time recursive least-squares (RLS) estimator was implemented in MATLAB Simulink to achieve a best fit from the measured angles in real time. The implementation of the least squares estimation follows that presented in [11]. The covariance matrix was initialized as the identity matrix, and the forgetting factor was set to unity.

B. Estimation of Crank Angle

Either of the independent angles (the knee angle or the IMU orientation) used for the link length estimation can be used to find the crank angle. In each case, however, the mapping is both multi-valued and, at certain points, ill-conditioned. First consider the mapping from γ, which is the supplementary angle for the quantity $(\theta_k - \theta_p)$, to φ_c.

$$\varphi_c = \varphi_0 - \arccos\left(\frac{r_c^2 + r_0^2 - r_p^2 - r_t^2 + 2 r_p r_t \cos\gamma}{2 r_c r_0}\right) \quad (3)$$

The inverse cosine ($y = \arccos x$) is typically defined over the principal domain of $\{-1 \leq x \leq 1\}$ and range of $\{0 \leq y \leq \pi\}$ to avoid ambiguity. The crank angle, however, must be allowed to evolve from 0 to 2π, and so (3) alone will be insufficient for calculating φ_c.

φ_c can also be written as a function of φ_p,

$$\varphi_c = 2\arctan\left(\frac{-B_p \pm \sqrt{B_p^2 - 4A_pC_p}}{2A_p}\right) \quad (4)$$

where the coefficients A_p, B_p, and C_p are nonlinear functions of the link lengths and φ_p.

Even if a four-quadrant inverse tangent is applied, the quadratic expression still yields two possible values for φ_c. Consequently, (4) is also multi-valued. Within a reasonable tolerance, however, one output from each expression should be in agreement, resolving the ambiguity. The estimation algorithm therefore continuously evaluates the four conditions and selects the two closest values of φ_c as the most likely estimates for each expression.

Using two estimates of φ_c not only resolves the multi-valued problem, but it also provides an opportunity to minimize the errors resulting from singularities in either estimate. A linear combination of the two estimates is constructed using normalized weights calculated from the relative magnitudes of the derivatives of (3) and (4). Although explicit differentiation of (3) and (4) is difficult, the derivatives can be expressed implicitly as

$$\frac{d\varphi_c}{d\gamma} = -\frac{r_t r_p \sin\gamma}{r_0 r_c \sin(\varphi_0 - \varphi_c)} \tag{5}$$

and

$$\frac{d\varphi_c}{d\varphi_p} = -\frac{r_p \sin(\varphi_p - \varphi_t)}{r_c \sin(\varphi_c - \varphi_t)} \tag{6}$$

respectively. The weights for the γ and φ_p estimates, respectively, are given by

$$G_\gamma = 1 - \frac{(\frac{d\varphi_c}{d\gamma})^2}{(\frac{d\varphi_c}{d\gamma})^2 + (\frac{d\varphi_c}{d\varphi_p})^2} \tag{7}$$

$$G_{\varphi_p} = 1 - \frac{(\frac{d\varphi_c}{d\varphi_p})^2}{(\frac{d\varphi_c}{d\gamma})^2 + (\frac{d\varphi_c}{d\varphi_p})^2}. \tag{8}$$

The weights as functions of the output are plotted in the bottom graph of Fig. 2. As each derivative approaches infinity, its respective contribution to the crank estimate approaches zero. Note also that, in general, the derivative of the output with respect to the knee angle measure is smaller than the derivative with respect to the shank angle measure, causing the knee measure to dominate the estimation except near its singularities.

III. Validation

A powered knee and ankle prosthesis previously developed by the authors was fitted to a bicycle and connected to an artificial passive hip joint mounted to the seat post. The internal signals of the prosthesis (knee angle and shank orientation) were logged simultaneously with external motion capture. The data from the prosthesis were streamed to MATLAB Simulink (running Real-time Windows Target) via a Controller Area Network (CAN) interface at a rate of 250 Hz. In Simulink, an ankle torque reference was computed from the position and velocity signals and returned to the prosthesis embedded system in order to emulate a stiff spring and damper system. With no significant external torques applied, the ankle remained fixed while the author moved the bicycle crank through forward and backward rotation using the contralateral crank arm. A torque reference of zero was applied to the knee joint in order to allow it to move freely.

In the motion capture software environment, five rigid bodies were defined with reflective markers corresponding to the bicycle frame, crank arm, prosthetic foot, prosthetic shank, and artificial thigh. The markers were tracked with a 12 camera motion capture package from Natural Point at 120 fps. These data were then exported to MATLAB for post processing. Principle component analysis was performed on the set of all marker locations for each joint axis. The mean direction of the third principle components of all the axes was used to reduce the data to 2 dimensions. The resulting data were then used to compute the link lengths and angles of the mechanism. A photograph of the setup, along with the motion capture model determined by the markers, is shown in Fig. 3.

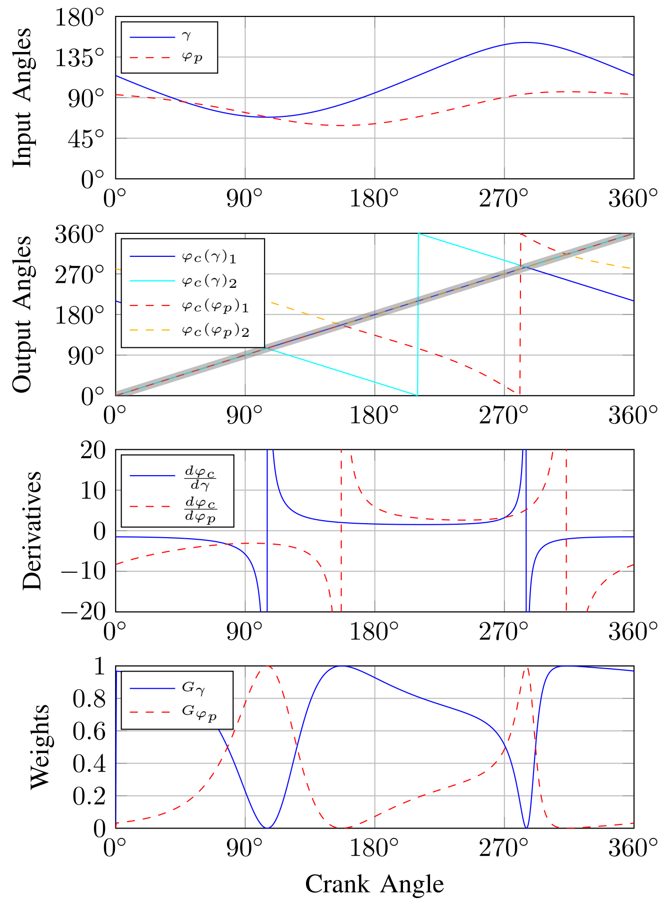
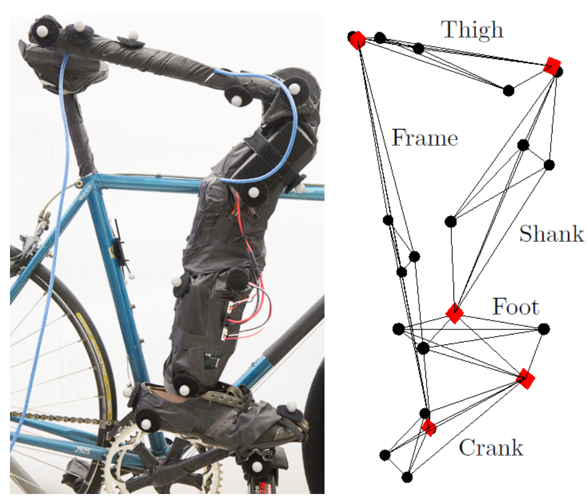


Fig. 2: Theoretical plots of ideal four bar behavior. The input angles were generated from the inverses of (3) and (4) as φ_c was swept from 0 to 360°. The second plot shows both possible values of φ_c from (3) and (4), along with the true value of φ_c. The third plot shows how $d\varphi_c/d\gamma$ and $d\varphi_c/d\varphi_p$ evolve as functions of φ_c, indicating the two singularities in the inverse mappings. The fourth plot shows the weights G_γ and G_{φ_p} used to combine the matching outputs of (3) and (4) to generate the estimate of φ_c.



(a) photograph (b) motion capture model

Fig. 3: The powered prosthesis configured on the bicycle (a) and the motion capture model (b). The red squares in (b) denote the joint axes.

A. Parameter Estimation

Upon startup, the algorithm must first obtain sufficient data from the prosthesis in order to converge on the Freudenstein coefficients. A plot of the coefficient estimates is shown in Fig. 4. A comparison of the link lengths as determined by both the motion capture system and the prosthesis is provided in Table I.

TABLE I: Determination of four-bar link lengths

Link	Motion Capture		Prosthesis	
	μ (m)	σ (m)	μ (m)	σ (m)
r_0	0.751	0.002	0.732	0.006
r_t	0.335	0.003	0.319	0.005
r_p	0.611	0.001	0.610	0.000
r_c	0.167	0.008	0.163	0.003

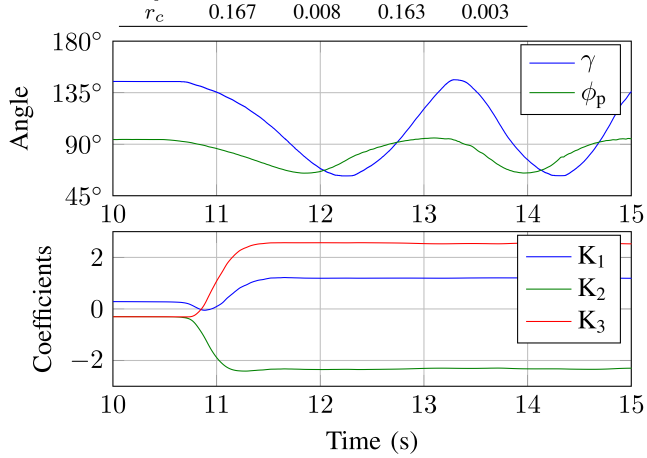


Fig. 4: Convergence of Freudenstein coefficients using RLS.

B. Crank Angle Estimation

The error in the crank angle measure over three cycles is shown in Fig. 5. Also included in Fig. 5 are the angular errors for each independent estimate of the crank angle before their linear combination. Note that the effect of the singularities is clearly present in each signal, and also that these errors are reduced using the fusion technique described. The measured and estimated crank angles from an entire trial including forward and backward rotation are plotted as functions of time in Fig. 6. In this trial the crank was moved in both directions. The maximum error after parameter convergence in this trial was approximately 20° due to a direction reversal (at 19 s).

IV. Conclusion

The presented algorithm avoids singularities from both measurements for the geometry used in the experiment, and estimates the crank angle within 15° when the hip joint is constrained and the bicycle is driven in the forward direction. Future work will include using the estimate to time the delivery of knee torque in order to supplement an amputee subject's effort during steady-state cycling.

Acknowledgment

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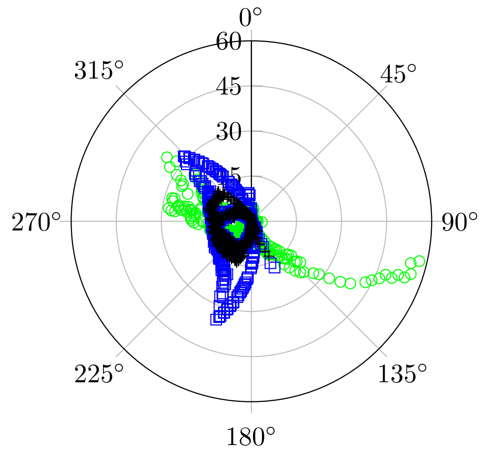

Fig. 5: Error as a function of crank angle. The error from the γ-based estimate is denoted by blue squares, the error from the φ_p-based estimate by green circles, and the combined estimate by black crosses.

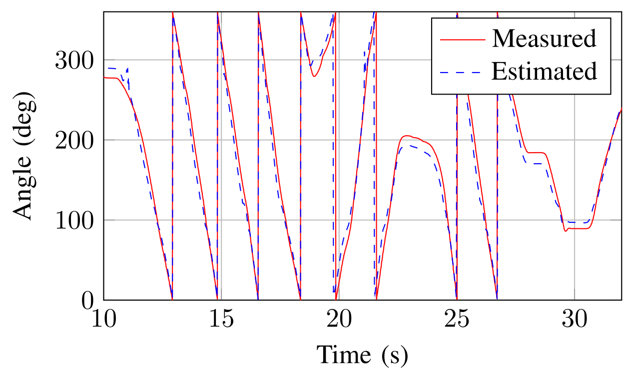

Fig. 6: Crank angle as measured by the motion capture system and estimated by the prosthesis for the trial showing forward and backward rotation.

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