Assessment of Gait Nonlinear Dynamics by Inhomogeneous Point-Process Models

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***Abstract*— Point-process linear models of stride intervals have been recently proven to provide a unique characterization of human gait dynamics through instantaneous time domain features. In this study we propose novel instantaneous measures characterizing nonlinear gait dynamics using a quadratic autoregressive inhomogeneous point-process model recently devised for the instantaneous assessment of physiological, natural, and physical discrete dynamical systems. Our mathematical framework accounts for long-term information given by the past events of non-stationary non-Gaussian time series, expressed by a Laguerre expansion of the Wiener-Volterra terms. Here, we present a study of gait variability from data gathered from physionet.org, including 15 recordings from young and elderly healthy volunteers, and patients with Parkinson's disease. Results show that our instantaneous polyspectral characterization provides an informative tracking of the inherent nonlinear dynamics of human gait, which is significantly affected by aging and locomotor disabilities.**

I. INTRODUCTION

The study of gait dynamics has provided important information on the physiology of human gait and on the locomotor control system, as well as objective measurement of mobility and functional status [1]. Indeed, dysfunctions of the neural control of locomotion, aging, and chronic diseases seriously affect gait variability [2], [3]. For instance, it has been shown that gait variability dramatically increases in patients with Parkinson's disease (PD) and Huntington's disease [2].

Recently, using linear statistical modeling, we demonstrated the intuitive point-process nature of human gait dynamics [4]. Each stride, in fact, can be represented by a sequence of *events*, and the stochastic series uncertainty refers to the timing of such events. Because the process can be seen as a sequence of zeros (absence of events) and ones (presence of events), this series can be effectively studied using the mathematical formulation given by point process theory [5]–[10]. The intrinsic probabilistic structure of each stride event generation is defined as function of the previous strides. As the probability function of a stride event is defined at each moment in time, point-process modeling allows to continuously characterize the discrete dynamics in an instantaneous fashion. Goodness-of-fit measures are also provided via a Kolmogorov-Smirnov (KS) test derived from the time-rescaling theorem [5]. Other successful applications of the point process methodology concern a very wide range of phenomena such as earthquake occurrences [11], traffic modeling [12], neural spiking activity [13], human heartbeats dynamics [5]–[9], [14]. Preliminary point-process applications to gait variability have allowed us to define instantaneous measures of stride dynamics [4]. As a step forward, we now investigate nonlinear gait dynamics. Several studies, in fact, demonstrated that stride pace is the result of complex neural and somato-motor activities generating moment-to- moment oscillations with significant nonlinear and fractal properties [15]–[17]. As a matter of fact, nonlinear measures such as multifractal indices [16], [17] and Lyapunov exponents [15] have been revealed as powerful quantifiers of gait dynamics. Accordingly, the aim of this study is related to the application of nonlinear point-process models for the estimation of gait nonlinear dynamics and extraction of novel instantaneous measures. We took advantage from our recent work in which we defined a novel inhomogeneous point-process framework endorsed by second-order nonlinear autoregressive terms, allowing for instantaneous estimation of the signal polyspectra, such as the dynamic bispectrum and trispectrum [6], [8], [9]. This nonlinear framework includes long-term memory capability through the Laguerre expansions of the Wiener-Volterra kernels [6], [7], [9], [18]. The orthonormal basis of the discrete-time Laguerre functions expands the kernels and reduces the number of unknown parameters that need to be estimated.

Here, we present four mathematical formulations specifically tailored for modeling gait dynamics: a quadratic Nonlinear fully Autoregressive (NAR) model, and a NAR model with Laguerre expansion of kernels (NARL), along with their two linear counterparts (AR and ARL, respectively). Remarkably, using the NAR and NARL models it is possible to define novel instantaneous quantifiers such as the polyspectral power of the stride variability. Experimental results are obtained by applying all four models to data of stride times (available on PhysioNet [19]) gathered from 15 recordings from young and elderly healthy volunteers, and patients with Parkinson's disease.

II. METHODOLOGY OF STATISTICAL SIGNAL PROCESSING

Let us consider the expectation of the Taylor expansion of a general NAR model formulation driven by independent, identically distributed Gaussian random variables with zero

The research leading to these results has received partial funding from the Department of Anesthesia, Critical Care & Pain Medicine, Massachusetts General Hospital, Harvard Medical School, Boston (MA), USA.

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mean [6]:

$$E[y(k)] = \gamma_0 + \sum_{i=1}^{M} \gamma_1(i)\, y(k-i) + \sum_{n=2}^{\infty} \sum_{i_1=1}^{M} \cdots \sum_{i_n=1}^{M} \gamma_n(i_1, \ldots, i_n) \prod_{j=1}^{n} y(k-i_j)\ . \quad (1)$$

Given the autoregressive structure of eq. 1, the system can be identified with only exact knowledge of the output data. This formulation can be embedded into the point-process framework and used to model the first order moment of the inter-event probability of the locomotor systems considering nonlinear terms up to the second order, i.e. γ_0, $\gamma_1(i)$, and $\gamma_2(i,j)$. In order to reduce the number of unknown parameters that need be estimated in eq. 1, it is possible to expand the linear and nonlinear terms by using the Laguerre functions [18]:

$$\phi_j(k) = \alpha^{\frac{k-j}{2}} (1-\alpha)^{\frac{1}{2}} \sum_{i=0}^{j} (-1)^i \binom{k}{i} \binom{j}{i} \alpha^{j-i} (1-\alpha)^i$$

where ϕ_j is the j^{th}-order discrete time orthonormal Laguerre function with $k \geq 0$, and α is the discrete-time Laguerre parameter ($0 < \alpha < 1$) which determines the rate of exponential asymptotic decline of these functions.

A. Point-Process Model of Gait Nonlinear Dynamics

Given a single event E and the events set $\{u_j\}_{j=1}^{J}$ detected from the stochastic time series, $\mathrm{EE}_j = u_j - u_{j-1} > 0$ denotes the j^{th} E-E interval within the an observation interval $t \in (0, T]$ [6]. Assuming history dependence and an inverse Gaussian probability distribution of the waiting time $t - u_j$ until the next E, it is possible to write:

$$f(t|\mathcal{H}_t, \xi(t)) = \left[\frac{\xi_0(t)}{2\pi(t-u_j)^3}\right]^{\frac{1}{2}} \times \exp\left\{-\frac{1}{2} \frac{\xi_0(t)[t - u_j - \mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))]^2}{\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))^2 (t-u_j)}\right\} \quad (2)$$

with $j = \widetilde{N}(t)$ the index of the previous event before time t and $\widetilde{N}(t)$ the left continuous sample path of the associated counting process, $\mathcal{H}_t = (u_j, \mathrm{EE}_j, \mathrm{EE}_{j-1}, ..., \mathrm{EE}_{j-M+1})$,$\xi(t)$ the vector of the time-varying parameters, $\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))$ the first-moment statistic (mean) of the distribution, and $\xi_0(t) > 0$ the shape parameter of the inverse Gaussian distribution. As $f(t|\mathcal{H}_t, \xi(t))$ indicates the probability of having an event at time t given that a previous event has occurred at u_j and $\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))$ can be interpreted as signifying the most probable moment when the next event is likely to occur.

B. Linear and Nonlinear Modeling of $\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))$

In this study, we define two nonlinear models, along with their two linear counterparts, by characterizing the mean of the IG probability function within the point-process framework. The four models are defined as follows:

- The NAR model with degree of nonlinearity 2 as follows:

$$\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t)) = \mathrm{EE}_{\widetilde{N}(t)} + \sum_{i=1}^{p} \gamma_1(i,t)\, \Delta\mathrm{EE}_{\widetilde{N}(t)-i} + \gamma_0 + \sum_{i=1}^{q} \sum_{j=1}^{q} \gamma_2(i,j,t)\, \Delta\mathrm{EE}_{\widetilde{N}(t)-i}\, \Delta\mathrm{EE}_{\widetilde{N}(t)-j} \quad (3)$$

  where $\Delta\mathrm{EE}_{\widetilde{N}(t)-i} = \mathrm{EE}_{\widetilde{N}(t)-i} - \mathrm{EE}_{\widetilde{N}(t)-i-1}$.
- The AR linear model is defined considering only terms up to the first order in the eq. 3.
- The NARL formulation, i.e. a NAR using also the Laguerre expansion of the Wiener-Volterra terms, is as follows:

$$\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t)) = \mathrm{EE}_{\widetilde{N}(t)} + g_0(t) + \sum_{i=0}^{p} g_1(i,t)\, l_i(t^-) + \sum_{i=0}^{q} \sum_{j=0}^{q} g_2(i,j,t)\, l_i(t^-)\, l_j(t^-)\ . \quad (4)$$

  where $l_i(t^-) = \sum_{n=1}^{\widetilde{N}(t)} \phi_i(n)(\mathrm{EE}_{\widetilde{N}(t)-n} - \mathrm{EE}_{\widetilde{N}(t)-n-1})$ is the output of the Laguerre filters just before time t. Note that the Laguerre expansion of the g_0,$\{g_1(i)\}$, and $\{g_2(i,j)\}$ allows to retain all the past information of the series, even with a finite degree of nonlinearities.
- The ARL linear model is defined considering only terms up to the first order in eq. 4.

Of note, in all formulations we have considered the derivative E-E series in order to improve stationarity within the sliding time window W, and we have chosen $W = 90$ sec. in all cases [6], [8]. Because $\mu_{\mathrm{EE}}(t, \mathcal{H}_t, \xi(t))$ is defined in continuous time, it is possible to obtain instantaneous estimates at very fine, arbitrary timescales, without interpolation between the arrival times of two events. Given a local observation interval $(t-l, t]$ of duration l, we find the unknown time-varying parameter vector $\xi(t) = [\xi_0(t), g_0(t), g_1(0,t), ..., g_1(p,t), g_2(0,0,t), ..., g_2(i,j,t)]$. that maximizes the local log-likelihood through the well-known Newton-Raphson procedure [5], [6]. The recursive, causal nature of the estimation allows to predict each new observation given the previous history. The model and all its parameters are therefore also updated at each iteration without priors. We determine the optimal order $\{p, q\}$ based on the Akaike Information Criterion and the model goodness-of-fit (obtained by prefitting the model to a subset of the data), which is based on the Kolmogorov-Smirnov (KS) test and associated KS statistics [5], [6]. Autocorrelation plots are also considered to test the independence of the model-transformed intervals [5]. Once the order $\{p, q\}$ is determined, the initial NARL coefficients are estimated by the method of least squares [5], [6].

C. Novel Quantitative Tools of Gait Dynamics: Linear and Nonlinear Variance from Instantaneous Polyspectra

In order to provide quantitative tools related to polyspectra representations, it is necessary to link the NAR and NARL models to the traditional input-output Wiener-Volterra model [6], [8]. The transformation between eq. 3 and the input-output Wiener-Volterra model can be performed in the frequency domain by using the following general relationship [6] between the Fourier transforms of the input-output Volterra kernels of order p, $H_p(f_1, \ldots, f_n)$, to be estimated and the Fourier transforms of the extended NAR terms, $\Gamma_1'(f_1)$ and $\Gamma_2'(f_1, f_2)$ [6]:

$$\sum_{p=\mathrm{mid}(q)}^{q} \sum_{\sigma \in \sigma_q} H_p(f_{\sigma(1)}, ..., f_{\sigma(r)}, \omega_{\sigma(r+1)} + f_{\sigma(r+2)}, ..., f_{\sigma(q-1)} + f_{\sigma(q)}) \times \Gamma_1'(f_{\sigma(1)}) \cdots \Gamma_1'(f_{\sigma(r)}) \times \Gamma_2'(f_{\sigma(r+1)}, f_{\sigma(r+2)}) \cdots \Gamma_2'(f_{\sigma(q-1)}, f_{\sigma(q)}) = 0 \quad (5)$$

where q is a given integer representing the kernel order, $\mathrm{mid}(q) = \lceil q/2 \rceil$, $r = 2p - q$ and σ_q is the permutation set of N_q.

As quantitative dynamical features, in this study we consider the instantaneous estimation of the first and second order moments (μ_{EE} and σ_{EE}), the spectral power $\mathcal{Q}(t)$, and the bispectral power $B(t)$. The spectral power is derived from the time-varying linear autospectrum computed as:

$$\mathcal{Q}(t) = \int_f 2(1 - cos(\omega)) S_{xx}(f,t) H_1(f,t) H_1(-f,t) - \frac{3}{2\pi} \int H_3(f, f_2, -f_2, t) S_{xx}(f_2, t) df_2 \quad (6)$$

where $S_{xx}(f,t) = \sigma^2_{\mathrm{EE}}$.

The bispectral power is derived by the dynamical Bispectrum as reported in [6] by integrating $|B(f_1, f_2)|$ in the appropriate triangular region of non-redundancy.

III. Experimental Data and Results

We tested the performance of our linear and nonlinear point process models in experimental datasets (from Physionet.org) comprised of ~300 consecutive strides gathered from 15 subjects: 5 healthy young (age 23-29), 5 healthy elderly (age 60-77), and 5 PD patients (age 71-77) [19]. In these datasets, gait data (i.e., stride time intervals such that each event E is represented by a stride) was recorded via an ultra-thin force-sensitive switch placed in the insole of the right shoe while subjects walked around a large oval track (>160 m), and logged via a recorder strapped to the right ankle [19].

Exemplary plots of the stride times, gait variability, histograms, first and second order NARL terms, as well as the bispectra, are shown in Fig. 1. All data were processed in order to evaluate the KS statistics and perform a further comparison between the four models. KS distance values from all of the subjects and for each model are reported in Table I. This analysis revealed more emphasized gait nonlinear dynamics in both elderly subjects and PD patients. In young subjects, in fact, linear models provided the best fitting performance on all subjects, whereas the nonlinear models provided significantly lower KS distances on the other two groups.

Then, we tested if our indices (i.e. the first ($\mu_{\mathrm{EE}_{\mathrm{GAIT}}}(t, \mathcal{H}_t, \xi(t))$) and second moment order ($\sigma_{EE_{GAIT}}$), as well as the spectral ($Q(t)$) and bispectral ($B(t)$) power) were able to uniquely chracterize each of the three groups (i.e. Young, Elderly, PD) . All the features were extracted

TABLE I KS distances from the Young, Elderly, and PD subjects datasets using point-process linear and nonlinear models.

YOUNG	NARL	NAR	ARL	AR
Subj 1	0.0439	0.0676	0.0459	**0.0350**
Subj 2	0.0343	0.0349	0.0272	**0.0214**
Subj 3	0.0257	0.0380	0.0238	**0.0213**
Subj 4	0.0389	0.0528	**0.0296**	0.0411
Subj 5	0.0371	0.0466	**0.0248**	0.0379

ELDERLY	NARL	NAR	ARL	AR
Subj 1	**0.0205**	0.0461	0.0301	0.0250
Subj 2	0.0395	**0.0333**	0.0468	0.0341
Subj 3	**0.0241**	0.0426	0.0280	0.0359
Subj 4	**0.0171**	0.0218	0.0235	0.0233
Subj 5	**0.0164**	0.0326	0.0228	0.0291

PD	NARL	NAR	ARL	AR
Subj 1	0.0524	**0.0515**	0.0969	0.0671
Subj 2	**0.0416**	0.05312	0.0744	0.0704
Subj 3	0.0469	**0.0373**	0.0574	0.0487
Subj 4	**0.0976**	0.1313	0.1199	0.1245
Subj 5	**0.0617**	0.1259	0.0790	0.0993

Bold values indicate the best result fit.

TABLE II KS distances from the Young, Elderly, and PD subjects datasets using point-process linear and nonlinear models.

	Young	Elderly	PD
μ_{EE}(ms)	1113.5 $\pm$ 69.3	1032.7 $\pm$ 75.7	1132.3 $\pm$ 105.5
σ_{EE} (ms^2) *	267.58 $\pm$ 27.12	204.23 $\pm$ 32.48	2343.0 $\pm$1025.6
$Q(t)^*$	255.22 $\pm$ 32.58 (Low)	172.40 $\pm$ 30.20 (Low)	2962.9 $\pm$ 1749.9 (High)
$B(t)^*$	186.16 $\pm$ 31.66 (High)	33.48 $\pm$ 2.56 (Low)	145.06 $\pm$ 37.46 (High)

* $p < 0.05$ from non-parametric Kruskall-Wallis test with null hypothesis of equal medians

from the NARL point-process model and were averaged for each recording. Results, obtained through the Kruskall-Wallis test with null hypothesis of equal medians, are shown in Table II and expressed as $\mathrm{Median}(X) \pm \mathrm{MAD}(X)$, where $\mathrm{MAD}(X) = \mathrm{Median}(|X - \mathrm{Median}(X)|)$. We found that no statistical difference was associated to the $\mu_{\mathrm{EE}_{\mathrm{GAIT}}}(t, \mathcal{H}_t, \xi(t))$ feature exclusively, whereas the other three NARL features were associated to a $p < 0.01$. In particular, a further post-hoc analysis through the Mann-Whitney non-parametric test revealed that the second moment order $\sigma_{EE_{GAIT}}$ and the spectral power $Q(t)$ resulted significantly higher in patients with PD than young and elderly subjects ($p < 0.03$), whereas the bispectral power $B(t)$ resulted significantly lower in elderly subjects than young subjects and PD ($p < 0.01$). Therefore, all three groups are unequivocally identified. The PD group is identified by high spectral and bispectral powers, the Elderly group by low spectral and bispectral powers, and the Young group by low spectral and high bispectral powers (see Table II).

IV. Conclusion and Discussions

In this study, we test the applicability of the NAR and NARL inhomogeneous point-process models to human gate data. By extending the previous point process methodological

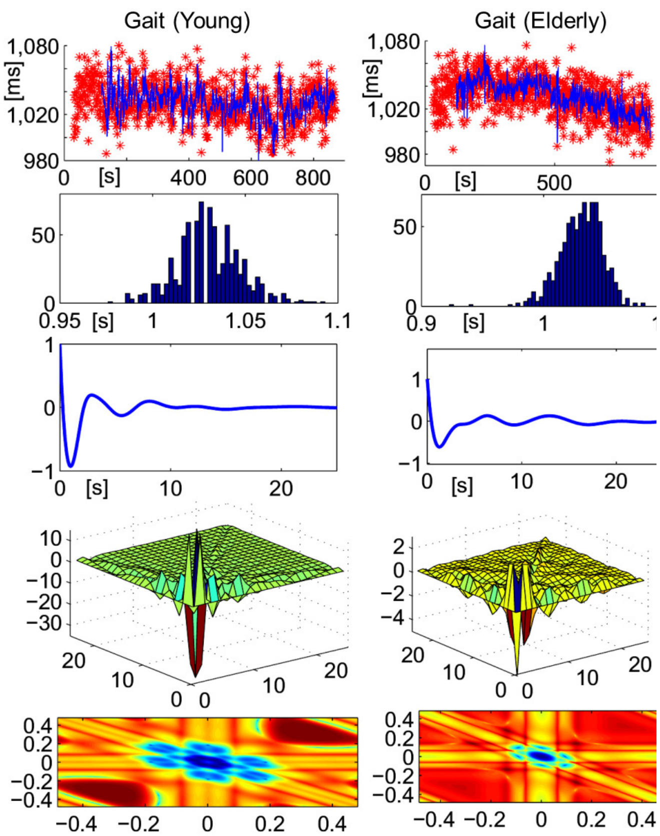

Fig. 1. Plots related (from the left) a gait of a young person and gait of an elderly person. (from the top) In the first panel, the estimated NARL $\mu(t)$, shown in blue, is superimposed on the recorded series (red asterisks). Below, the histograms, the first and the second order NARL terms, and the bispectra are shown. On the bottom, the KS distance values for the considered nonlinear models, i.e. NARL and NAR, are reported.

approach based on linear modeling [4], we investigate gait nonlinear dynamics in young and elderly subjects, as well as in PD patients. Our nonlinear probabilistic framework inherits all the advantages of the previous linear models [5] and, thanks to the novel fully nonlinear autoregressive formulation, is able to provide a novel nonlinear characterization through estimates of polyspectra. Of note, these features cannot be simply computed as the Fourier transform of the second and high-order input-output transfer functions, because they also embed higher-order nonlinear feedbacks [6], [9], [14].

In the presented application, we demonstrate that nonlinear models provide a better description of gait dynamics in elderly subjects and patients with PD, whereas a linear structure is more prevalent in young subjects (see Tables I), possibly reflecting stronger and healthier control mechanisms of locomotion. In addition, we have been able to univocally discern the short walkings gathered from Young, Elderly and PD. In particular, PD patients are defined by highest spectral power, Elderly subjects by highest bispectral power, whereas only a combination of spectral and bispectral power is able to separate young subjects from both Elderly and PD (see Table II). The presented novel instantaneous features of gait nonlinear dynamics can open interesting avenues to explore the dynamics of human gait in real-time, including correlations with other behavioral and physiological measures, possibly in a clinical scenario. Future work focuses on the application of other time-varying nonlinear and complexity measures (e.g. entropy), also applied to different groups of subjects and patients with locomotor disabilities.

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